M.Sc. (CA) (2nd Semester)

040020206: Computer Oriented Operations Research Mehtods

Question Bank

Unit: 1

Introduction of Operations Research and Linear Programming

Q: 1 Short Answer Questions:

- 1. Write the definition of operation research.
- 2. Write the definition of solution, basic solution.
- 3. What is Linear programming problem?
- **4.** How will you construct a mathematical model.
- **5.** Write down any two scopes of operation research.
- **6.** Describe the phase formulating the problem.
- 7. Define i] Solution ii] Basic solution.
- 8. Define feasible solution.
- 9. Define LPP in the mathematical form.
- **10**. Give any four models of operations research.
- 11. List different activities of marketing.
- **12.** List different activities of personnel management.
- **13.** What is model?
- 14. State any four application of OR.
- 15. Advantages of OR study.
- **16.** Disadvantages of OR study.
- 17. Define unbounded solution.
- 18. Define feasible and Infeasible solution.
- 19. What is extreme point?
- **20.** Draw graph for $3a + 5b \ge 20$, Where $a,b \ge 0$.
- **21.** State the different scope of operation research.
- 22. What are various phases of operation research?
- 23. What is model? List out the model.

Q: 2 Long Answer Questions.

- 1. A firm manufactures two products A and b on which the profits earned per unit are Rs 3 and Rs 4 respectively. Each product is processed on two machines M1 and M2. Product A requires one minute of processing time on M1 and two minutes on M2 while B requires one minute on M1 and one minute on M2. Machine M1 is available for not more than 7 hours, while machine M2 is available for 10 hours during any working day. Formulate the number of units of products A and B to be manufactured to get maximum profit.
- 2. The ABC Company has been a producer of picture tubes for television sets and certain printed circuits for radios. The company has just explained into full scale production and marketing of AM and AM-FM radios. It has built a new plant that can operate 48 hours per week. Production of an AM radio in the new plant will require 2 hours and production of an AM-FM radio will require 3 hours. Each AM radio will contribute Rs. 40 to profits while an AM-FM radio will contribute Rs. 80 to

profits. The marketing departments, after extensive research, have determined that a maximum of 15 AM radios and 10 AM-FM radios can be sold each week. Formulate the LPP.

3. Sudhakant has two iron mines. The production capacities of the mines are different. The iron ore can be classifies into good, mediocre and bad varieties after certain process. The owner has decided to supply 12 or more tons of good iron, 8 or more tons of mediocre iron and 24 or more tons of bad iron per week. The daily expense is Rs.2000 and that of the second mine is Rs.1600. The daily production of each type of iron is given in the table.

| Mine | Daily production | | | | | |
|------|------------------|----------|-----|--|--|--|
| | Good | Mediocre | Bad | | | |
| 1 | 6 | 2 | 4 | | | |
| II | 2 | 2 | 12 | | | |

Formulate the LPP.

- **4.** Write the steps for solving Linear Programming Problem by Graphical method. State its limitations
- 5. Solve the following LP problems graphically

Minimize
$$Z = 3 x_1 + 2 x_2$$

Subject to

$$5 x_1 + x_2 \ge 10,$$

 $x_1 + x_2 \ge 6,$

$$x_1 + 4 \quad x_2 \geq 12$$

$$x_1$$
 , $x_2 \ge 0$

6. Solve the following LP problems graphically

Maximize
$$Z = 30 \times 1 + 20 \times 2$$

Subject to

$$3x_1 + 3x_2 \ge 40$$

$$2x_1 + 5x_2 \geq 44$$

$$x_1$$
 , $x_2 \ge 0$

7. Find the solution using graphical method

Max
$$z = 3x_1 + 5x_2$$

$$x_1+2x_2 \le 2000$$

$$x_1 + 2x_2 \le 1500$$

8. Solve the following LP graphically:

Maximize $z = 8000x_1 + 7000x_2$

Subject to

$$3x_1 + x_2 \le 66$$

$$x_1 + x_2 \le 45$$

$$x_1 \leq 20$$

$$x_2 \le 40$$

$$x_1, x_2 \ge 0$$

- **9.** Explain the history of operations research.
- 10. Write the algorithm to solve LPP using Graphical method for maximization of profit.
- **11.** Give the limitations of operations research.
- **12.** Note down the applications of operations research.
- 13. Write down meanings of operations research.
- **14.** Solve the following LP graphically:

Maximize
$$z = x_1 + x_2$$

Subject to

$$x_1 + 2x_2 \le 2000$$

 $x_1 + x_2 \le 1500$ $x_2 \le 600$ $x_1, x_2 \ge 0$ 15. Solve LP graphically Maximize $z = 300x_1 + 400x_2$ Subject to $5x_1 + 2x_2 \le 180$ $3x_1 + 3x_2 \le 135$ $x_1, x_2 \ge 0$ 16. Solve LP graphically Max $Z = 40x_1 + 30x_2$ Subject to $3x_1 + x_2 \le 30$

- $x_1 \le 8$
- $x_2 \le 12$
- and $x_1, x_2 \ge 0$
- 17. Discuss various phases in solving an OR problem.
- **18.** Explain the steps involoved in the solution of an operation research problem.
- 19. Define O.R. and discuss its scope.
- 20. Discuss the inportence of OR in decision making.

Q: 3 Multiple choice questions.

- 1. Operation research approach is
 - A) Multi-disciplinary

B) Artificial

C) Intuitive

- D) All of the above
- **2.** Operation research analysis does not
 - A) Predict future operation

B) Build more than one model

C) Collect the relevant data

- D) Recommended decision and accept
- 3. Mathematical model of Linear Programming is important because
 - A) It helps in converting the verbal description and numerical data into mathematical expression
 - B) decision makers prefer to work with formal models.
 - C) it captures the relevant relationship among decision factors.
 - D) it enables the use of algebraic techniques.
- 4. A constraint in an LP model restricts
 - A) value of the objective function
- B) value of the decision variable
- C) use of the available resourses
- D) all of the above
- 5. In graphical method of linear programming problem if the ios-cost line coincide with a side of region of basic feasible solutions we get
 - A) Unique optimum solution
- B) unbounded optimum solution

- C) no feasible solution
- D) Infinite number of optimum solutions

- 6. A feasible solution of LPP
 - A) Must satisfy all the constraints simultaneously
 - B) Need not satisfy all the constraints, only some of them
 - C) Must be a corner point of the feasible region
 - D) all of the above
- 7. The objective function for a L.P model is 3x1+2x2, if x1=20 and x2=30, what is the value of the objective function?

A) 0

B) 50

C) 60

D) 120

- 8. Maximization of objective function in LPP means
 - A) Value occurs at allowable set decision
 - B) highest value is chosen among allowable decision
 - C) none of the above
 - D) all of the above
- 9. Alternative solution exist in a linear programming problem when
 - A) one of the constraint is redundant
 - B) objective function is parallel to one of the constraints
 - C) two constraints are parallel
 - D) all of the above
- **10.** Linear programming problem involving only two variables can be solved by
 - A) Big M method
 - B) Simplex method
 - C) Graphical method
 - D) none of these
- 11. The linear function of the variables which is to be maximize or minimize is called
 - A) Constraints
 - B) Objective function
 - C) Decision variable
 - D) None of the above
- 12. A physical model is an example of
 - A) An iconic model
 - B) An analogue model
 - C) A verbal model
 - D) A mathematical model
- 13. If the value of the objective function z can be increased or decreased indefinitely, such solution is called
 - A) Bounded solution
 - B) Unbounded solution
 - C) Solution
 - D) None of the above
- 14. A model is
 - A) An essence of reality
 - B) An approximation
 - C) An idealization
 - D) All of the above
- **15.** The first step in formulating a linear programming problem is
 - A) Identify any upper or lower bound on the decision variables
 - B) State the constraints as linear combinations of the decision variables
 - C) Understand the problem
 - D) Identify the decision variables
- **16.** Constraints in an LP model represents
 - A) Limititations
 - B) Requirements
 - C) balancing limitations and requirements
 - D) all of above
- 17. The best use of linear programming is to find optimal use of
 - A) money
 - B) Manpower
 - C) Machine

- D) All of the above
- **18.** Which of the following is assumption of an LP model
 - A) divisibility
 - B) proportionality
 - C) additivity
 - D) all of the above
- **19.** Before formulating a formal LP model, it is better to
 - A) Express each constraints in words
 - B) Express the objective function in words
 - C) verbally identify decision variables
 - D) all of the above
- 20. Non-negative condition in an LP model implies
 - A) a positive coefficient of variables in objective function
 - B) a positive coefficient of variables in any constraint
 - C) non-negative value of resourse
 - D) none of the above

Q: 4 True or false.

- 1. Adding a constraint to a linear programming problem increases the size of the feasible region.
- **2.** The following constraint is linear: $A*B+2*A \le 20$.
- 3. The two objective functions (Maximize 5X + 7Y, and Minimize -5X-7Y) will produce the same solution to a linear programming problem.
- 4. Redundancy causes major difficulties to an LP problem.
- **5.** All linear programs must seek to maximize some quantity.
- 6. When we say that proportionality exists, we mean that if one unit of a product takes two units of a resource, then three units of the product will take three units of the resource.
- 7. If the right-hand side of a constraint is changed, the feasible region will not be affected and will remain the same.
- 8. One of the quickest ways to plot a constraint is to find the two points where the constraint crosses the axes, and draw a straight line between these points.
- 9. If the objective function coefficients are slightly changed within certain limits, the current optimal solution may remain optimal.
- **10.** Changes in the technological coefficients affect the shape of the objective function.
- 11. The first step in formulating a linear programming problem should be to identify the objective and constraints.
- 12. Linear programming is a model consisting of linear relationships representing a firm's decisions given an objective and resource constraints.
- **13.** The objective function is a linear relationship reflecting the objective of an operation.
- 14. A linear programming model consists of decision variables, constraints, but no objective function.
- **15.** A feasible solution violates at least one of the constraints.
- 16. In a linear programming model, the number of constraints must be less than the number of decision variables.
- 17. Linear programming models have an objective function to be maximized but not minimized.
- 18. Linear programming models exhibit linearity among all constraint relationships and the objective
- 19. The graphical approach to the solution of linear programming problems is a very efficient means of solving problems.
- **20.** Slack variables are only associated with maximization problems.
- **21.** Surplus variables are only associated with minimization problems.

Q: 5 Fill in the blanks.

| 1. The graphical method can only be used when there are decision variables. |
|---|
| 2. The term implies that one or more variables in the solution and the profit can be |
| infinitely large. |
| 3. When the slope of the objective function is the same as the slope of one or more constraints, we |
| may have |
| 4. To find the optimal solution to an LP problem when using the graphical method, we must first |
| dentify the |
| 5. The problem involves a decision as to which products to produce given limited |
| resources. |
| 6. The is that region which satisfies all constraints. |
| 7. The represents all points that result in a specified profit. |
| 8. An important function of sensitivity analysis is to |
| 9. Isoprofit method involves graphing profit lines. |
| 10 . The for a constraint is the value of one additional unit of a scarce resource. |
| 11 is the first step in formulating an LP problem. |
| 12. LP theory states that the optimal solution to any problem will lie at |
| 13. A of a system of two equations in two variables is an ordered pair that makes both |
| equations true. |
| 14. A of a system of linear inequalities is an ordered pair that is a solution of each |
| nequality in the system. |
| 15. The graph of the system of constraints in linear programming is called the |
| 16. is the process of optimizing a linear objective function subject to a system of linear |
| nequalities. |
| 17. A graphical solution is limited to solving linear programming problems with only decision |
| variables. |
| 18. The optimal solution to a linear programming model always occurs at a (an) point of |
| the feasible region. |
| 19. Multiple optimal solutions can occur when the objective function line is to a |
| constraint line. |
| 20. The property of linear programming models indicates that the decision |
| variables cannot be restricted to integer values and can take on any fractional value. |
| |

Unit: 2

Simplex Method for Solution of LPP

Q: 1 Short answer question.

- 1. Define surplus variables.
- 2. What is the criterion for the entering variable and outgoing variable?
- **3.** Define artificial variables.
- 4. Define slack variables.
- **5.** When is Big M method useful?
- **6.** What is the condition for optimality in simplex table?
- 7. What is the condition for entering variable in simplex table?
- **8.** Write the standard form of LPP for the following LPP:

Maximize
$$Z = 13x_1 + 25x_2$$

Subject to
$$21x_1 + 3x_2 \le 40$$
, $5x_1 + 2x_2 \le 7$, $x_1, x_2 \ge 0$

9. Write the standard form of LPP for the following LPP:

Maximize
$$Z = 3x_1 + 5x_2$$

Subject to
$$2x_1 + 3x_2 \le 4$$
, $3x_1 + 2x_2 \ge 7$, $x_1, x_2 \ge 0$

- **10.** What is the artificial variable?
- **11.** List assumption of linear programming.
- 12. Limitations of Linear Programming
- **13.** Write the steps of the algorithm for solving LPP by the Simplex method.
- **14.** What does the Cj Zj row represent in a simplex tableau?
- **15.** What shadow price indicates in a simplex final solution?
- **16.** How an unbounded solution is recognized when calculating the simplex tableaus.
- 17. How a multiple optimal solution is recognized when using the simplex algorithm.
- **18.** How no feasible solution is recognized when using the simplex algorithm.
- 19. How degeneracy is recognized when using the simplex algorithm.

Q: 2 Long answer questions:

- 1. Explain about Big –M method for solving LPP by the Simplex method.
- **2.** Max $z = 18 x_1 + 24 x_2$

Subject to

$$4 x_1 + 2 x_2 \le 8$$

$$2 x_1 + 5x_2 \le 12$$

$$x_1$$
 , $x_2 \ge 0$

3. Max
$$z = 30 x_1 + 20 x_2$$

Subject to

$$3 x_1 + x_2 \ge 40$$

$$2 x_1 + 5 x_2 \ge 44$$

$$x_1$$
 , $x_2 \ge 0$

4. Maximize
$$Z = 3x_1 + 5x_2$$

Subject to

$$\mathsf{x}_1+\mathsf{x}_2\leq \mathsf{4},$$

$$3x_1+2x_2\leq 18$$

$$x_1$$
 , $x_2 \ge 0$

5. Maximize $Z = 3 x_1 + 2x_2$

Subject to

$$2x_1+x_2\leq 5,$$

$$X_1 + x_2 \le 3$$
,

$$X_1, x_2 \ge 0$$

6. Maximize $Z = 3 x_1 + 4x_2$

Subject to

$$x_1+x_2\leq 6,$$

$$2X_1 + 4x_2 \le 20$$
,

$$X_1, x_2 \ge 0$$

7. Maximize $Z = 7x_1 + 5x_2$

Subject to

$$x_1 + 2x_2 \le 6,$$

$$4x_1 + 3x_2 \le 12$$

$$x_1$$
 , $x_2 \ge 0$

8. Solve by Big M method

Maximize
$$Z = 3x_1 - x_2$$

Subject to $2x_1 + x_2 \ge 2$, $x_1 + 3x_2 \le 3$, $x_1, x_2 \ge 0$
9. Solve by Big M method Maximize $Z = 3x_1 - x_2 + 4x_3$
Subject to $3x_1 - x_2 - x_3 \le -2$, $5x_1 + 2x_2 \ge -3$, $x_1, x_2, x_3 \ge 0$
10. Solve the following LP problem using simplex method. Maximize $z = 3x_1 + 2x_2 + 5x_3$
Subject to $x_1 + 2x_2 + x_3 \le 430$ $3x_1 + 2x_3 \le 460$ $x_1 + 4x_2 \le 420$ $x_1, x_2, x_3 \ge 0$
11. Use penalty (Big M) method to maximize: $z = 3x_1 - x_2$
Subject to $2x_1 + x_2 \ge 2$ $x_1 + 3x_2 \le 3$ $x_2 \le 4$

12. Solve by simplex method:

 $x_1, x_2 \ge 0$

Min
$$z = x_1 - 3x_2 + 2x_3$$

 $3x_1 - x_2 + 3x_3 \le 7$
 $-2x_1 + 4x_2 \le 12$
 $-4x_1 + 3x_2 + 8x_3 \le 10$
 $x_1, x_2, x_3 \ge 0$

13. Solve by simplex method:

Max z =
$$3x_1 + 5x_2 + 4x_3$$

Subject to
 $2x_1 + 3x_2 \le 8$
 $2x_2 + 5x_3 \le 10$
 $3x_1 + 2x_2 + 4x_3 \le 15$

 $x_1, x_2, x_3 \ge 0$

14. A manufacturer of three products tries to follow a policy of producing those which continue most to fixed cost and profit. However, there is also a policy of recognising certain minimum sales requirements currently, these are:

Product: x_1 \mathbf{X}_{2} X_3 Units per week: 20 30 60

There are three producing departments. The production times in hour per unit in each department and the total times available each week in each department are:

| Departments | Time required per p | Total | hours | | | | | | |
|-------------|---------------------|----------------|----------------|-----------|--|--|--|--|--|
| | x_1 | X ₂ | X ₃ | available | | | | | |
| 1 | 0.25 | 0.20 | 0.15 | 420 | | | | | |
| 2 | 0.30 | 0.40 | 0.50 | 1048 | | | | | |
| 3 | 0.25 | 0.30 | 0.25 | 529 | | | | | |

The contribution per unit of product x_1 , x_2 , x_3 is Rs. 10.50, Rs. 9.00 and Rs. 8.00 respectively. The company has scheduled 20 units of x_1 , 30 units of x_2 and 60 units of x_3 for production in the following week, you are required to state:

- 1) Whether the present schedule is an optimum one from a profit point of view and if it is not, what it should be;
- 2) The recommendations that should be made to the firm about their production facilities.
- **15.** Use the simplex method solve the following LP problem :

```
Max Z = 30x + 40y + 20z

subject to

10x + 12y + 7z \le 10,000

7x + 10y + 8z \le 8,000

x + y + z \le 1,000

x, y, z \ge 0
```

- **16.** A manufacturer of bags makes three types of bags P, Q and R which are processed on three machines M1, M2 and M3. Bag P requires 2 hours on machine M1 and 3 hours on machine M2 and 2 hours on machine M3. bag Q requires 3 hours on machine M1, 2 hours on machine M2 and 2 hours on machine M3 and Bag R requires 5 hours on machine M2 and 4 hours on machine M3. There are 8 hours of time per day available on machine M1, 10 hours of time per day available on machine M2 and 15 hours of time per day available on machine M3. The profit gained from bag P is Rs 3.00 per unit, from bag Q is Rs 5.00 per unit and from bag R is Rs 4.00 per unit. What should be the daily production of each type of bag so that the products yield the maximum profit?
- 17. Comment on the solution of the following LP problem:

```
Max Z = -2x_1 + 3x_2
subject to
         x_1 \le 5
         2x_1 - 3x_2 \le 6
         x_1, x_2 \ge 0.
18. Comment on the solution of the following LP problem:
Min Z = 2x_1 + 8x_2
subject to
         5x_1 + x_2 \ge 10
         2x_1 + 2x_2 \ge 14
         x_1 + 4x_2 \ge 12
         x_1, x_2 \ge 0.
19. Comment on the solution of the following LP problem:
  Max Z = 3x_1 + 2x_2
subject to
         2x_1 + x_2 \le 2
         3x_1 + 4x_2 \le 12
         x_1, x_2 \ge 0.
20. Comment on the solution of the following LP problem:
Max Z = 3x_1 + 2x_2 + 3x_3
subject to
         2x_1 + x_2 + x_3 \le 2
         3x_1 + 4x_2 + 2x_3 \le 8
         x_1, x_2, x_3 \ge 0.
```

Q: 3 Multiple Choice Questions:

1. The role of artificial variables in the simplex method is

(a) to aid in finding an initial solution

| (b) to find optimal dual prices in the fina | l simplex table. |
|---|--|
| (c) to start with Big M method. | |
| (d) to start with Big M method. | |
| 2. For a maximization problem, the objective fur | nction coefficient for an artificial variable is |
| (a) + M (b) –M (c) Zero | (d) None of these |
| 3. For maximization LPP, the simplex method is t | erminated when all values |
| (a) $cj - zj \le 0$ (b) $cj - zj \ge 0$ (c) $cj - zj$ | $=0 		 (d) zj \leq 0$ |
| 4. To convert ≥ inequality constraints into equali | ty constraints, we must |
| (a) add a surplus variable | |
| (b) subtract an artificial variable | |
| (c) subtract a surplus variable and an ad- | d artificial variable |
| (d) add a surplus variable and subtract a | n artificial variable. |
| 5. If any value in b - column of final simplex tabl | e is negative, then the solution is |
| (a) unbounded | (b) infeasible |
| (c) optimal | (d) None of these |
| 6. To convert ≥ inequality constraints into equali | ty constraints, we must |
| (a) add a surplus variable | |
| (b) subtract an artificial variable | |
| (c) subtract a surplus variable and an ad- | d artificial variable |
| (d) add a surplus variable and subtract a | n artificial variable. |
| 7. At every iteration of simplex method, for mir | nimization problem, a variable in the current basis is |
| replaced with another variable that has | |
| (a) a positive cj −zj value | (b) a negative cj −zj value |
| (c) $cj - zj = 0$ | (d) None of these |
| 8. In the optimal simplex table $cj - zj = 0$ value in | dicates |
| (a) unbounded solution | (b) cycling |
| (c) alternative solution | (d) None of these |
| 9. A variable which does not appear in the basis | variable column of simplex table is |
| | (b) always equal to zero |
| | (d) None of these |
| · | implex method, we must add artificial variable to |
| (a) only equality constraints | (b) only > constraints |
| | (d) None of these |
| 11. If all xij values in the incoming variable colun | |
| (a) solution is unbounded | (b) there are multiple solution |
| (c) there exist no solution | (d) None of these |
| • | c variable column of optimal simplex table, then the |
| solution is | |
| (a) unbounded | (b) infeasible |
| (c) optimal | (d) None of these |
| 13. If for a given solution, a slack variable is equa | |
| (a) the solution is optimal | (b) the solution is infeasible |
| (c) there exist no solution | (d) None of these |
| 14. Linear programming problem involving more | |
| (a) Simplex method | (b) Graphical method |
| (c) Matrix minima method | (d) None of these |
| 15. In the simplex method, a tableau is optimal of | |
| (a) zero or negative. | (b) zero. |
| (c)negative and nonzero. | (d) positive and nonzero. |
| 16. For the basic feasible solution to remain opti | mal |
| (a)all cj - zj values must remain ≤ 0. | |

- (b)no objective function coefficients are allowed to change.
- (c) the value of the objective function must not change.
- (d) each of the above is true.
- 17. Consider the following linear programming problem.

Maximize 40 X₁ + 30 X₂ + 60 X₃ Subject to: $X_1 + X_2 + X_3 \le 90$ $12 X_1 + 8 X_2 + 10 X_3 \le 1500$ $X_1 = 20$ $X_3 \le 100$ $X_1, X_2, X_3 \ge 0$

How many slack, surplus, and artificial variables would be necessary if the simplex algorithm were used to solve this problem?

- (a) 3 slack, 6 surplus, and 6 artificial
- (b) 2 slack, 1 surplus, and 2 artificial
- (c) 1 slack, 2 surplus, and 2 artificial
- (d) 1 slack, 2 surplus, and 1 artificial
- **18.** The following are constraints which have had slack variables added.

$$2 X_1 + 4 X_2 + S1 = 20$$

 $6 X_1 + 4 X_2 + S2 = 36$

If $X_1 = 2$ and $X_2 = 2$, what are the values for S1 and S2?

(a) S1 = 20, S2 = 36

(b) S1 = 16, S2 = 32

(c) S1 = 8, S2 = 16

(d) S1 = 0, S2 = 0

- 19. A solved LP problem indicated that the optimal solution was X_1 =10 and X_2 =20. One of the constraints was $4 X_1 + 2 X_2 \le 80$. This constraint has
 - (a) surplus greater than zero.
- (b) slack greater than zero.

(c) surplus equal to zero.

- (d) slack equal to zero.
- **20.** The solution to the dual LP problem
 - (a) presents the marginal profits of each additional unit of a resource.
 - (b) can always be derived by examining the Zj row of the primal's optimal simplex tableau.
 - (c) is better than the solution to the primal.
 - (d) all of the above

Q: 4 True or False

- 1. A basic feasible solution is a solution to a linear programming problem that corresponds to a corner point of the feasible region.
- 2. A surplus variable is added to $a \ge constraint$ in order to create equality, and represents a quantity of unused resource.
- **3.** A surplus variable is added to an = constraint in order to utilize the simplex algorithm.
- 4. If all of a resource represented by a slack variable is used, that slack variable will not be in the production mix column of a linear programming simplex tableau.
- 5. Even if an LP problem involves many variables, an optimal solution will always be found at a corner point of the n-dimensional polyhedron forming the feasible region.
- 6. A correctly formulated linear program, when solved with the simplex algorithm, will always yield a single optimal solution.
- 7. Unlike the Solver algorithm, Simplex will only produce a single solution even if multiple solutions
- **8.** Surplus variables, like slack variables, carry a zero cost.

- **9.** The constraint $5X_1 + 6X_2 \ge 30$, when converted to an = constraint for use in the simplex algorithm, will be $5X_1 + 6X_2 S + A = 30$.
- **10.** The constraint 5 X_1 + 6 $X_2 \le 30$, when converted to an = constraint for use in the simplex algorithm, will be 5 X_1 + 6 X_2 S = 30.
- **11.** The constraint 5 X_1 + 6 X_2 = 30, when converted to an = constraint for use in the simplex algorithm, will be 5 X_1 + 6 X_2 + M = 30.
- **12.** Linear programming has few applications in the real world due to the assumption of certainty in the data and relationships of a problem.
- **13.** Typically, real world applications of linear programming are solved with a computer program that utilizes the simplex algorithm.
- **14.** The basic process of the simplex algorithm is to find solutions to a set of simultaneous equations where we have more variables than equations.
- **15.** It is possible for an equation in the simplex table to have both a slack and a surplus variable at the same time.
- **16.** In the simplex table, a coefficient of zero in a constraint implies that the variable with the zero coefficient has no influence on the solution for that iteration.
- **17.** The simplex method considers both feasible and infeasible solutions.
- **18.** The simplex method finds a solution by solving for the intersection of two constraints.
- 19. When the optimal solution is found, all slack and surplus variables have a value of zero.
- **20.** The numbers in the body of the simplex table may be thought of as the solutions.

| 0 | • | 5 | Fi | II | in | th | 16 | h | lan | ks. |
|---|---|---|----|----|----|----|----|---|-----|-----|
| | | | | | | | | | | |

| 4.61 |
|---|
| 1. Simplex method was developed by in 1947. |
| 2. According to fundamental theorem of linear programming, we can search the solution |
| among the basic feasible solutions. |
| 3. If a constraint has a sign ≤, in order to convert it into an equation we use variables. |
| 4. If a constraint has a ≥ sign then to convert it into an equation we use variables. |
| 5. A linear programming problem is said to have an solution if the objective function can be |
| increased or decreased indefinitely. |
| 6. To convert the problem of minimization into the maximization problem we multiply both sides by |
| 7. To solve a L.P.P. by simplex method all b _i 's should be |
| · |
| 8. If in the simplex table all $\Delta_j \leq 0$, the solution under test is |
| 9. The linear programming problem has no solution if the solution contains one or more |
| artificial variables as basic variables. |
| 10. If in the final simplex table all Δ_j < 0, the optimal solution is |
| 11. If corresponding to maximum positive Δ_j all minimum ratios are negative or $\rightarrow \infty$, the solution under test is |
| 12. A slack variable is introduced if the given constraint has sign. |
| 13. Simplex method to solve linear programming problems was developed by |
| 14. Fundamental theorem of L.P.P. states that if the given L.P.P. has an optimal solution then at |
| least basic solution must be optional. |
| 15. In phase 1 of two phase method we remove from the basic matrix. |
| 12 D = = = |

| 16. The problem has solution if the value of at least one artificial variable present in the |
|---|
| basis is non-zero and the optimality condition is satisfied. |
| 17. To prepare a less-than-or-equal constraint for use in a simplex table, we must add a(n) |
| variable. |
| 18. To prepare a greater-than-or-equal constraint for use in a simplex table, we must add |
| - |
| |

Unit: 3

Transportation and Assignment Problem (TP & AP)

Q: 1 Short answer questions.

- 1. What is Transportation problem?
- 2. State the mathematical formulation for T.P.
- 3. What is an Assignment Problem?
- 4. What is the difference between Assignment Problem and Transportation Problem?
- 5. State the mathematical formulation for A.P.
- **6.** What is the Travelling salesman problem?
- 7. Write steps for North-West Corner Method.
- 8. Write steps for Matrix Minima Method.
- 9. What is difference between transportation problem and assignment problem?
- 13. Write the steps for solving a A.P. by Hungarian method
- **14.** Write the steps for transportation algorithm.
- 15. Define feasible solution.
- **16.** What is basic feasible solution?
- **17.** Define optimal solution.
- **18.** State different methods to obtain the initial basic feasible solution.
- **19.** What are the properties of non-degenerate solution?
- **20.** How do the MODI and stepping-stone methods differ?

Q:2 Long answer questions.

1. Three fertilizers factories X, Y and Z located at different places of the country produce 6,4 and 5 lakh tones of urea respectively. Under the directive of the central government, they are to be distributed to 3 States A, B and C as 5, 3 and 7 lakh respectively. The transportation cost per tones in rupees is given below:

| | Α | В | С |
|---|----|----|----|
| Χ | 11 | 17 | 16 |
| Υ | 15 | 12 | 14 |
| Z | 20 | 12 | 15 |

Find out suitable transportation pattern at minimum cost by North West Corner method and Least Cost method.

2. Determine an IBFS by Vogel's Approximation method.

| | , , | | | | |
|--------|-----|----|----|----|--------|
| Source | D1 | D2 | D3 | D4 | Supply |
| S1 | 19 | 30 | 50 | 10 | 7 |
| S2 | 70 | 30 | 40 | 60 | 9 |
| S3 | 40 | 8 | 70 | 20 | 18 |

|--|

3. A departmental has five employees with five jobs to be performed. The time (in hours) each men will take to perform each job is given in the effectiveness matrix.

How should the jobs be allocated, one per employee, so as to minimize the total man-hours . **Employees**

| jobs | 1 | 2 | 3 | 4 | 5 |
|------|----|----|----|----|----|
| а | 10 | 5 | 13 | 15 | 16 |
| b | 3 | 9 | 18 | 13 | 6 |
| С | 10 | 7 | 2 | 2 | 2 |
| d | 7 | 11 | 9 | 7 | 12 |
| е | 7 | 9 | 10 | 4 | 12 |

4. A city corporation has decided to carry out road repairs on main four arteries of the city. The government has agreed to make a special grant of Rs 50 lakh towards the cost with a condition that repairs are done at the lowest cost and quickest time. If the conditions warrant, a supplementary token grant will also be considered favorably. The corporation has floated tenders and five contractors have sent in their bids. In order to expedite work, one road will be awarded to only one contractor.

Cost of Repairs (Rs in lakh)

| | • | | | | |
|-------------|----|----|----|----|----|
| Contractors | R1 | R2 | R3 | R4 | R5 |
| C1 | 9 | 14 | 19 | 15 | 13 |
| C2 | 7 | 17 | 20 | 19 | 18 |
| C3 | 9 | 18 | 21 | 18 | 17 |
| C4 | 10 | 12 | 18 | 19 | 18 |
| C5 | 10 | 15 | 21 | 16 | 15 |

Find the best way of assigning the repair work to the contractors and the costs.

If it is necessary to seek supplementary grants, what should be the amount sought?

5. A company has factories at F1, F2 & F3 which supply warehouses ay W1,W2 and W3. Weekly factory capacities are 200,160 and 90 units respectively. Weekly warehouses requirements are 180,120 and 150 units respectively. Unit shipping costs (in rupees) are as follows:

Warehouse

| Factory | w1 | w2 | w3 | Supply |
|---------|-----|-----|-----|--------|
| F1 | 16 | 20 | 12 | 200 |
| F2 | 14 | 8 | 18 | 160 |
| F3 | 26 | 24 | 16 | 90 |
| Demand | 180 | 120 | 150 | |

Determine the optimum distribution for this company to minimize the shipping cost.

6. Obtain the initial solution to above TP using northwest corner method.

| | D1 | D2 | D3 | D4 | Supply |
|------|----|----|----|----|--------|
| 01 | 6 | 4 | 1 | 5 | 14 |
| 02 | 8 | 9 | 2 | 7 | 16 |
| 03 | 4 | 3 | 6 | 2 | 5 |
| Dem. | 6 | 10 | 15 | 4 | |

7. Obtain the initial solution to bellow TP using least cost method.

| | Α | В | С | D | Supply |
|--------|---|----|----|---|--------|
| 1 | 6 | 3 | 5 | 4 | 22 |
| II | 5 | 9 | 2 | 7 | 15 |
| III | 5 | 7 | 8 | 6 | 8 |
| Demand | 7 | 12 | 17 | 9 | |

8. Obtain the initial solution to Below TP using least cost method.

| | | D1 | D2 | D3 | D4 | Supply | |
|-----|----------------|------------|------------|----------|-----------|-------------------------|-------|
| | 01 | 1 | 2 | 3 | 4 | 6 | |
| | 02 | 4 | 3 | 2 | 0 | 8 | |
| | 03 | 0 | 2 | 2 | 1 | 10 | |
| | Demand | 4 | 6 | 8 | 6 | | |
| | | | | | | | |
| 9. | | | | | | | |
| | | Α | В | С | D | Supply | |
| | 1 | 1 | 5 | 3 | 3 | 34 | |
| | II | 3 | 3 | 1 | 2 | 15 | |
| | III | 0 | 2 | 2 | 3 | 12 | |
| | IV | 2 | 7 | 2 | 4 | 19 | |
| | | 21 | , 25 | 2 17 | 4 17 | 19 | |
| | Demand | | | | | | |
| 40 | Obtain the ini | itiai soiu | tion to a | above 1P | using n | orthwest corner method | • |
| 10. | | | | | | | |
| | _ | l . | II | Ш | IV | Supply | |
| | Α | 21 | 16 | 15 | 13 | 11 | |
| | В | 17 | 18 | 14 | 23 | 13 | |
| | С | 32 | 27 | 18 | 41 | 19 | |
| | Demand | 6 | 10 | 12 | 15 | | |
| | Obtain the ini | itial solu | tion to a | above TP | using V | ogel's approximation me | thod. |
| 11. | | | | | | | |
| | | D1 | D2 | D3 | D4 | Supply | |
| | 01 | 1 | 2 | 1 | 4 | 30 | |
| | 02 | 3 | 3 | 2 | 1 | 50 | |
| | 03 | 4 | 2 | 5 | 9 | 20 | |
| | Demand | 20 | 40 | 30 | 10 | | |
| | | | | | | ogel's approximation me | thod |
| 12. | Obtain the in | iciai soia | | | u3.11.6 v | oger o approximation me | tilou |
| 12. | | I | II | Ш | IV | Supply | |
| | ۸ | 2 | 3 | 11 | | | |
| | A B | 1 | 0 | 6 | 7 | 6 1 | |
| | | | | | 1 | | |
| | C | 5 | 8 | 15 | 10 | 10 | |
| | Demand | 7 | 5 | 3 | 2 | 17 | |
| | Obtain the op | otimal so | olution to | o above | TP. | | |
| 13. | | | | | | | |
| | | a | b | С | Suppl | У | |
| | I | 10 | 9 | 8 | 8 | | |
| | II | 10 | 7 | 10 | 7 | | |
| | III | 11 | 9 | 7 | 9 | | |
| | IV | 12 | 14 | 10 | 4 | | |
| | Demand | 10 | 10 | 8 | | | |
| | Obtain the ini | itial solu | tion to a | above TP | using n | orthwest corner method | |
| 14. | | houses | | - · | 6 | | |
| | | W1 | W2 | W3 | W4 | Supply | |
| | F1 | 19 | 30 | 50 | 10 | 3аррту 7 | |
| | F2 | 70 | 30 | 40 | 60 | 9 | |
| | ΓZ | 70 | 30 | 40 | 00 | 3 | |

Obtain the optimal solution to above TP .

15. Solve the following Assignment Problem

F3

Demand

| | Р | Q | R | S |
|---|----|----|----|----|
| Α | 22 | 30 | 21 | 15 |
| В | 18 | 33 | 9 | 31 |
| С | 44 | 25 | 24 | 21 |
| D | 23 | 30 | 28 | 14 |

16. Solve the following Assignment Problem

| | I | II | III | IV |
|---|----|----|-----|----|
| 1 | 11 | 10 | 18 | 5 |
| 2 | 14 | 13 | 12 | 19 |
| 3 | 5 | 3 | 4 | 2 |
| 4 | 15 | 18 | 17 | 9 |

17. Solve the following Assignment Problem

| | Α | В | С | D | E | F |
|---|----|----|----|----|----|----|
| 1 | 13 | 13 | 16 | 23 | 19 | 9 |
| 2 | 11 | 19 | 26 | 16 | 17 | 18 |
| 3 | 12 | 11 | 4 | 9 | 6 | 10 |
| 4 | 7 | 15 | 9 | 14 | 14 | 13 |
| 5 | 9 | 13 | 12 | 8 | 14 | 11 |

18. Solve the following Assignment Problem

```
Ρ
                           S
             Q
                    R
                           7
Α
      5
             3
                    4
В
      2
             3
                    7
                           6
С
             1
                    5
                           2
             8
                    1
                           2
```

19. Find the assignment of salesmen to various districts which will result minimum cost.

| alesman | | Distri | ct | |
|---------|----|--------|----|----|
| | 1 | 2 | 3 | 4 |
| Α | 16 | 10 | 14 | 11 |
| В | 14 | 11 | 15 | 15 |
| С | 15 | 15 | 13 | 12 |
| D | 13 | 12 | 14 | 15 |

20. Solve the following assignment problem so as to minimize the time (in days) required to complete all the task.

| person | | task | | | |
|--------|----|------|----|----|----|
| | 1 | 2 | 3 | 4 | 5 |
| Α | 6 | 5 | 8 | 11 | 16 |
| В | 1 | 13 | 16 | 1 | 10 |
| С | 16 | 11 | 8 | 8 | 8 |
| D | 9 | 14 | 12 | 10 | 16 |

21. An automobile dealer wishes to put four repairmen to four different jobs. The repairmen have somewhat different kinds of skills and they exhibit different levels of efficiency from one job to another. The dealer has estimated the number of manhours that would be required for each jobman combination. This is given in the matrix form.

Find the optimum assignment that will result in minimum manhours needed.

| | Α | В | С | D |
|---|---|---|---|---|
| 1 | 5 | 3 | 2 | 8 |
| 2 | 7 | 9 | 2 | 6 |
| 3 | 6 | 4 | 5 | 7 |
| 4 | 5 | 7 | 7 | 8 |

22. Solve the following transportation problem to find its optimal solution: (MODI)

| | W1 | W2 | W3 | W4 | Supply |
|--------|-----|-----|-----|-----|--------|
| P1 | 190 | 300 | 500 | 100 | 70 |
| P2 | 700 | 300 | 400 | 600 | 90 |
| Р3 | 400 | 100 | 400 | 200 | 180 |
| Demand | 50 | 80 | 70 | 140 | 340 |

23. Find initial basic feasible solution by 'Vogel's Method' and 'North West Corner Method'.

| | Destination | | | | | | |
|--------|----------------|----|----|----|----|----|--|
| | 1 2 3 4 supply | | | | | | |
| | 1 | 21 | 16 | 15 | 13 | 11 | |
| | П | 17 | 18 | 14 | 23 | 13 | |
| Source | Ш | 32 | 27 | 18 | 41 | 19 | |
| | Demand | 6 | 10 | 12 | 15 | 43 | |

24. Solve the following transportation problem to find its optimal solution: (MODI)

| | Α | В | С | Available |
|----------|-----|-----|-----|-----------|
| 1 | 50 | 30 | 220 | 1 |
| II | 90 | 45 | 170 | 3 |
| Ш | 250 | 200 | 50 | 4 |
| Required | 4 | 2 | 2 | |

Q: 3 Multiple Choice Questions:

1. The initial solution of a transportation problem can be obtained by applying any known method. However, the only condition is that

(a) the solution be optimal

(b)the rim condition are satisfied.

(c) the solution not be degenerate.

(d) all of the above.

2. The dummy source or destination in a transportation problem is added to

- (a) satisfyrim condition.
- (b) preventsolution from becoming degenerate.
- (c) ensure that total cost does not exceed a limit.
- (d) all of the above.
- 3. The occurrence of degeneracy while solving a transportation problem means that
 - (a) total supply equals total demand
- (b) the solution so obtained is not feasible.
- (c) the few allocations become negative.
- (d) none of the above.
- 4. An alternative optimal solution to a minimization transportation problem exists whenever opportunity cost corresponding to unused routes of transportation is:
 - (a) positive and greater than zero.
- (b) positive with at least one equal to zero.
- (c) negative with at least one equal to zero.
- (d) all of the above.
- 5. One disadvantage of using North-West Corner Rule to find initial solution to the transportation problem is that
 - (a) it is complicated to use.
 - (b) it does not take into account cost of transportation.
 - (c) it leads to degenerate initial solution
 - (d) all of the above.
- 6. The solution to a transportation problem with m-rows and n-columns is feasible if number of positive allocations are
 - (a) m+n

(b) m x n

(c) m + n - 1

- (d) all of the above.
- 7. The calculation of opportunity cost in the MODI method is analogous to a
 - (a) cj –zj value for non-basic variable columns in the simplex method.

| (b) value of a variable in b-c(c) variable in xb-column.(d) all of the above. | olumn of the simplex | method. |
|---|-------------------------|---|
| 8. If we were to use opportunity cos | t value for an unused | cell to test optimality, it should be |
| (a) equal to zero. | (| (b) most negative number. |
| (c) most positive number. | (| (d) all of the above. |
| | | |
| | | ase of a Transportation problem because |
| (a) the number of rows equ | | (b) all xij= 0 |
| (c) all rim conditions are 1 | | (d) all of above. |
| 10. For a salesman who has to visit i | n cities, following are | the ways of his tour plan |
| (a) n! | - | (b) (n + 1)! |
| (c) (n - 1)! | (| (d) n |
| | | t problem having a square pay-off matrix of |
| order n should have assignments eq | | |
| (a)2 n +1 | | (b) 2n-1 |
| (c) m+n+1 | | (d) M+n |
| 12. The purpose of a dummy row or | column in an assignm | ment problem is to |
| (a)obtain balance between | | |
| (b) prevent a solution from | | |
| (c)provide a means of repre | senting a dummy pro | blem |
| (d)none of the above. | | |
| 13. The Hungarian method for solvi | | bblem can also be used to solve |
| (a) a transportation problen | | |
| (b) a traveling salesman pro | blem | |
| (c) both (i) and (ii) | | |
| (d) only (ii) | | |
| 14. An optimal of an assignment pro | | |
| (a) each row and column ha | • | |
| (b) each row and column ha | | ement |
| (c) the data are arrangemen | t in a square matrix | |
| (d) non of the above | | |
| 15. The method used for solving an | | |
| (a) reduced matrix method | | (b) MODI method |
| (c) Hungarian method | | (d) none of the above |
| 16. Which method usually gives a ve | | |
| (a) northwest corner rule | | (b) Vogel's approximation method |
| (c) MODI method | • | (d) stepping-stone method |
| · | | ransportation model in which intermediate |
| transshipment points are be | | |
| (a) decreased | - | (b) deleted |
| (c) subtracted | - | (d) added |
| the | | ne transportation model method minimizes |
| (a) total demand | | (b) total supply |
| (c) total shipping cost | (| (d) number of destinations |
| 19. In an assignment problem, | | |
| (a) one agent can do parts o | | |
| (b) one task can be done by | _ | |
| (c) each agent is assigned to | its own best task | |
| (d) none of the above | | |

- 20. In a transportation problem, items are allocated from sources to destinations
 - (a) at a maximum cost

(b) at a minimum cost

(c) at a minimum profit

(d) at a minimum revenue

Q:4 True or False

- 1. Time series methods tend to be most useful for short-range forecasting.
- 2. An exponential smoothing forecast will react more strongly to immediate changes in the data than the moving average.
- 3. Longer-period moving averages react more quickly to recent demand changes than do shorterperiod moving averages.
- 4. The Delphi method for acquiring informed judgments and opinions from knowledgeable individuals uses a series of questionnaires to develop a consensus forecast about what will occur in the future.
- 5. Time series methods assume that what has occurred in the past will continue to occur in the future.
- 6. The operating characteristics of a queuing system provide information rather than an optimization of a queuing system.
- **7.** Queuing analysis is a deterministic technique.
- 8. A cereal company that is planning on marketing a new low-cholesterol cereal should be concerned about the states of nature--that is the probability that people will stay interested in eating healthy
- 9. The expected opportunity loss criterion will always result in the same decision as the expected value criterion.
- 10. Transportation roblems and assignment problems both of these problem types could not be solved using simplex algorithm.
- 11. Vogel's approximation method tackles the problem of finding a good initial solution by taking into account the costs associated with each route alternative.
- 12. The optimal solution of a transportation model provides a shipping pattern with the lowest total shipping cost.
- **13.** In a transportation problem, number of sources must be same as number of destinations.
- 14. In a transportation problem, the total demand of destinations must be identical to the total capacity of sources, otherwise QM cannot solve it.
- 15. In the assignment model, each person can be assigned to one and only one job.
- **16.** the assignment model, each job will be done by one and only one person.
- 17. The assignment problem can be viewed as a special transportation problem in which every source has capacity 1 and every destination has demand 1.
- 18. The transportation problem can be viewed as an assignment problem and solved by using the assignment model.
- 19. Demands of destinations is (are) the input(s) for solving a transportation problem.
- **20.** In the data-entering table of QM's transportation model, a source takes a row.

Q: 5 Fill in the blanks.

| In the data-entering table of QM's transportation model, a source takes a In the result of QM's transportation model, if it shows that Source 2 should ship "dummy" destination, then it means that | o 45 units to a |
|--|---------------------|
| 3. Dummy source or dummy destination will occur in the case of 4. In a transportation problem, we must make the number of and | equal. |
| | 19 Page |

| 5. The net cost of shipping one unit on a route not used in the current transportation problem |
|---|
| solution is called the |
| 6. The procedure used to solve assignment problems wherein one reduces the original assignment |
| costs to a table of opportunity costs is called |
| 7. The method of finding an initial solution based upon opportunity costs is called8 occurs when the number of occupied squares is less than the number of rows plus |
| the number of columns minus one. |
| 9. Both transportation and assignment problems are members of a category of LP problems called |
| 5. Both transportation and assignment problems are members of a category of Er problems called |
| 10. The equation $R_i + K_j = C_{ij}$ is used to calculate |
| 11. The transportation problem is to transport various amounts of a single homogenous commodity, |
| that are initially stored at various origins, to different destinations in such a way that the total |
| transportation cost is |
| 12. In a balanced m \times n transportation problem $\sum_{i=1}^{m} a_i =$, where a_i 's are capacities of the |
| sources and b_i s are requirements of the destinations. |
| 13. The transportation problem can be regarded as a generalization of the |
| 14. A feasible solution of m by n transportation problem is said to be non-degenerate basic solution |
| if number of positive allocations is exactly equal to |
| 15. By north-west corner rule we always get a basic feasible solution. |
| 16. The optimality test is applicable to a F.S. consisting of allocations in independent |
| positions. |
| 17. In a transportation problem the solution under test will be optimal and unique if all the cell |
| evaluations are |
| 18. In vogel's approximation method the differences of the smallest and second smallest costs in |
| each row and column are called |
| 19. In computational procedure of optimality test we choose that u_i or $v_j = 0$ for which the |
| corresponding row or column has the number of individual allocations. |
| 20. The iterative procedure of determining an optimal solution of a minimization transportation |
| problem is known as |
| |
| 11:4. / |
| Unit: 4 |
| Games Theory and Sequencing Problems |
| Q: 1 Short Answer Questions. |
| |
| 1. Define terms: Activity, Event, Merge Event, Burst Event, Total float, Free float, Critical path, critical |
| activity |
| 2. State Rules for Network Diagram. |
| 3. What is Sequencing problems? |
| 4. What is dummy activity? |
| 5. What is successor activity? |
| 6. Write similarities and differences between PERT and CPM. |
| 7. Write applications of PERT/CPM techniques. |
| 8. Write uses of PERT/CPM techniques. |
| 9. Write the steps for Processing n jobs through two machines. |
| 10. Define Competitive game |
| 11. Define payoff matrix |
| 12. Define pure Strategies |

- 13. Define Mixed Strategies
- 14. Define saddle point
- 15. Define optimal strategies
- **16.** What is two-person zero sum game? giving a suitable example.
- 17. Give minimax principle used in the theory of games.
- 18. Explain maximin principle used in the theory of games.
- 19. What is game in game theory?
- **20.** What are the properties of a game?
- 21. Explain the "best strategy" on the basis of minimax criterion of optimality.
- **22.** Write the notation of t_{ij} , T, I_{ij} .
- 23. Explain Number of Machines.
- 24. Define Processing Time.
- 25. Define Processing Order.
- **26.** Define Idle time on a Machine.
- 27. Define Total Elapsed Time.
- 28. Determine the value of the game.

| Player A | Player B | |
|----------------|----------------|----------------|
| | B ₁ | B ₂ |
| A ₁ | 2 | 6 |
| A ₂ | -2 | 4 |

29. Determine the value of the game.

| Player A | Player B | | |
|----------------|----------------|----------------|----------------|
| | B ₁ | B ₂ | B ₃ |
| A ₁ | -1 | 2 | -2 |
| A ₂ | 6 | 4 | -6 |

Q: 2 Long Answer Questions.

- 1. State the major limitations of the game theory.
- 2. Explain the difference between pure strategy and mixed strategy.
- 3. What are the assumptions made in the theory of game?
- 4. Explain four elements that characterize a sequencing problem.
- 5. Explain the principal assumptions made while dealing with sequencing problem.
- 6. Give three different examples of sequencing problems from your daily life.
- 7. Write a short note on the "sequencing decision problem for n jobs on two machines".
- 8. A company management and thelabour union are negotiating a anew three year settlement. Each of these has 4 strategies:

I: Hard and aggressive bargaining

II:- Reasoning and logical approach

III:- Legalistic strategy

IV:- Conciliatory approach

The cost to the company are given for every pair of strategy choice.

| Union | Company Strategies | | | |
|------------|--------------------|----|-----|----|
| Strategies | I | Ш | III | IV |
| I | 20 | 15 | 12 | 35 |
| II | 25 | 14 | 8 | 10 |
| III | 40 | 2 | 10 | 5 |
| IV | -5 | 4 | 11 | 0 |

9. Find the range of values of p and q that will render the entry a saddle point for the game.

| Player A | Player B | | |
|----------------|----------------|----------------|-------|
| | B ₁ | B ₂ | B_3 |
| A ₁ | 2 | 4 | 5 |
| A ₂ | 10 | 7 | q |
| A ₃ | 4 | Р | 6 |

10. For what value of λ , the game with following pay-off matrix is strictly determinable?

| | 0. | • | • | |
|----------|-------------------|---|----|--|
| Player A | Player B | | | |
| | B_1 B_2 B_3 | | | |
| A_1 | ٨ | 6 | 2 | |
| A_2 | -1 | λ | -7 | |
| A_3 | -2 | 4 | λ | |

11. Determine which of the following two-person zero-sum games are strictly determinable and fair. Give the optimum strategies for each player in the case of it being strictly determinable.

a)

| Player A | Player B | | | |
|----------|-------------|----|--|--|
| | B_1 B_2 | | | |
| A_1 | 1 | 2 | | |
| A_2 | 4 | -3 | | |

b)

| Player A | Player B | | |
|----------|----------|-------|--|
| | B_1 | B_2 | |
| A_1 | -5 | 2 | |
| A_2 | -7 | -4 | |

12. Solve the following games by using maximum principle, whose payoff matrix are given below: Include in your answer: (i) strategy selection for each player, (ii) the value of the game to each player. Does the game have a saddle point?

a)

| Player A | Player B | | | |
|----------|----------|-------|-------|-------|
| | B_1 | B_2 | B_3 | B_4 |
| A_1 | 1 | 7 | 3 | 4 |
| A_2 | 5 | 6 | 4 | 5 |
| A_3 | 7 | 2 | 0 | 3 |

b)

| Player A | | Player B | | | | | | | | | | | |
|----------|-------|----------|-------|-------|-------|--|--|--|--|--|--|--|--|
| | B_1 | B_2 | B_3 | B_4 | B_5 | | | | | | | | |
| A_1 | -2 | 0 | 0 | 5 | 3 | | | | | | | | |
| A_2 | 3 | 2 | 1 | 2 | 2 | | | | | | | | |
| A_3 | -4 | -3 | 0 | -2 | 6 | | | | | | | | |
| A_4 | 5 | 3 | -4 | 2 | 6 | | | | | | | | |

c)

| Player A | | Player B | | | | | | | | | |
|----------|-------|----------|-------|-------|--|--|--|--|--|--|--|
| | B_1 | B_2 | B_3 | B_4 | | | | | | | |
| A_1 | 3 | -5 | 0 | 6 | | | | | | | |
| A_2 | -4 | -2 | 1 | 2 | | | | | | | |
| A_3 | 5 | 4 | 2 | 3 | | | | | | | |

d)

| Player A | | Player B | | | | | | | | | |
|----------|-------|----------|-------|-------|--|--|--|--|--|--|--|
| | B_1 | B_2 | B_3 | B_4 | | | | | | | |
| A_1 | -5 | 3 | 1 | 10 | | | | | | | |
| A_2 | 5 | 5 | 4 | 6 | | | | | | | |
| A_3 | 4 | -2 | 0 | -5 | | | | | | | |

Q: 3 Multiple Choice Questions.

- 1. Two person zero-sum game means that the
 - (a) Sum of losses to one player is equal to the sum of gains to other
 - (b) Sum of losses to one player is not equal to the sum of gains to other
 - (c) Both (a) and (b)
 - (d) None of the above
- 2. Game theory models are classified by the
 - (a) Number of players

- (b) sum of all payoffs
- (c) Number of strategies
- (d) all of the above
- 3. A game is said to be fair if
 - (a) both upper and lower values of the game are same and zero
 - (b) upper and lower values of the game are not equal
 - (c) upper value is more than lower value of the game
 - (d) none of the above
- 4. What happens when maximin and minimax value of the game are same?
 - (a) no solution exists

(b) solution is mixed

(c) saddle point exists

(d) none of the above

- 5. A mixed strategy game can be solved by
 - (a) algebraic method

(b) matrix method

| | (c) graphical method | (d) all of the above |
|-----|---|---|
| 6. | The size of the payoff matrix of a game can be | e reduced by using the principle of |
| | (a) game inversion | (b) rotation reduction |
| | (c) dominance | (d) game transpose |
| 7. | The payoff value for which each player in a ga | ame always selects the same strategy is called the |
| | (a) saddle point | (b) equilibrium point |
| | (c) both (a) and (b) | (d) none of the above |
| 8. | Games which involves more than two players | s are called |
| | (a) conflicting game | (b) negotiable game |
| | (c) N-person game | (d) all of the above |
| 9. | - · · · | to the sum of losses to another player in a game, |
| | this situation is known as | , , |
| | | (b) zero-sum game |
| | (c) fair game | (d) all of the above |
| 10 | | matrix of a game, the value of the game is then found |
| | by | matrix of a game, the value of the game is then round |
| | | and column combination to calculate excepted |
| | payoff for that combination and adding all | |
| | (b) reducing size of the game to apply alge | |
| | (c) both (a) and (b) | istate method |
| | (d) none of the above | |
| 11 | . A saddle points exists when | |
| | (a) maximin value = maximax value | (b) minimax value = minimum value |
| | (c) minimax value = maximin value | (d) none of the above |
| 12 | . In a mixed strategy game | (d) Holle of the above |
| 12 | | (b) each player always selects same strategy |
| | • | gy without considering other player's choice |
| | (d) all of the above | gy without considering other player's choice |
| 12 | | |
| 13 | . In a pure strategy game | |
| | (a) any strategy may be selected arbitrarily | |
| | (b) a particular strategy is selected by each | • • |
| | (c) both player always selects their optima | ii strategy |
| 4.4 | (d) none of the above | |
| 14 | | to determine value of the game when size of payoff |
| | matrix is | / IV 2V |
| 4- | (a) 2X2 (b) 3X4 (c) mX2 | (d) 2Xn |
| 15 | . Game theory is the study of | (1) |
| | | (b) resolving conflict between players |
| | (c) both (a) and (b) | (c) none of the above |
| 16 | . If there are in jobs to be performed one at a | time, on each of m machines, the possible |
| | sequences would be | n |
| | (a) $(n!)^m$ (b) $(m!)^n$ (c) $(n)^m$ (d) $(m)^n$ | |
| 17 | . Total elapsed time to processes all jobs thro | |
| | (a) $\sum_{j=1}^{n} M_{1j} + \sum_{j=1}^{n} M_{2j}$ | (b) $\sum_{j=1}^{n} M_{2j} + \sum_{j=1}^{n} M_{1j}$ |
| | (c) $\sum_{j=1}^{n} (M_{1j} + I_{1j})$ | (d) none of the above |
| 18 | . The minimum processing time on machine $\it M$ | M_1 and M_2 are related as |
| | (a) Min T_{1j} = Max T_{2j} | (b) Min $T_{1j} \le \text{Max } T_{2j}$ |
| | (c) Min $T_{1j} \ge \text{Max } T_{2j}$ | (d)Max $T_{2j} \ge Min T_{1j}$ |
| 19 | . You would like to assign operators to the eq | , , |
| | | (b) job with the earliest due date |
| | | (d) all of the above |

- Department of Computer Science And Technology | 2015 20. Unforeseen factors the prevent the plans from actually happening are (a) equipment may develop a fault (b) specifications may be changed (c) additional order may arrive to be added to schedules (d) all of the above Q: 4 Fill in the blanks. 1. A game is said be _____ if lower and upper values of the game are same as well as zero. 2. A course of action that puts any player in the most preferred position irrespective of the course of action chosen by the competitor is called _____. 3. If a game involves more than two players, then it is called a _____ game. 4. The pure strategy on the only course of action that is _____ being chosen by a player. 5. The rules of are used to reduce the size of the payoff matrix. 6. The _____ is used to determine the probability of using different strategies by both the players. 7. The ______ is used to find the optimal strategy for each player in a payoff matrix of size 2X2 without saddle point. 8. The _____ is useful for game where payoff matrix is of size 2Xn or mX2. 9. The _____ method is useful for game with larger payoff matrix without saddle point. 10. The concept of _____ is used in simplifying the solution of a rectangular game. Q: 5 True of False. 1. In a row person zero-sum game, a saddle point always exists. 2. If there are only two strategies, the payoff matrix has a saddle point. 3. In a pure strategy game, each player always plays just one strategy. 4. In a mixed strategy game, each player random by chooses the strategy to be used. 5. A mixed strategy game is based on the assumption that players act irrationally. 6. The strategy for a player is a course of action that the adopts for each playoff. 7. The value of the game is the expected outcome per play when players follow their optimal strategy.
 - 8. The payoff is a quantitative measure of satisfaction that a player gets at the end of the play.
 - 9. If the maximin value is same as the minimax value, than the game is said to have a saddle point.
 - 10. A game said to be strictly determinable if lower and upper values of the game are equal.

Unit: 5

Project Scheduling (CPM and PERT)

Q: 1 Short Answer Questions.

- 1. Define terms: Activity, Event, Merge Event, Burst Event, Total float, Free float, Critical path, critical
- 2. State Rules for Network Diagram.
- **3.** What is Sequencing problems?
- **4.** What is dummy activity?
- **5.** What is successor activity?
- 6. Write similarities and differences between PERT and CPM.
- 7. Write applications of PERT/CPM techniques.

- 8. Write uses of PERT/CPM techniques.
- **9.** Write the steps for Processing n jobs through two machines.

Q: 2 Long Answer Questions.

1. Draw the Network Diagram for the following activities and find the critical path.

| Job | Α | В | С | D | Ε | F | G | Τ | ı | J | K |
|------------------------|----|---|----|---|----|----|-----|---|---|-----|----|
| Job time(days) | 13 | 8 | 10 | 9 | 11 | 10 | 8 | 6 | 7 | 14 | 18 |
| Immediate predecessors | - | Α | В | С | В | Ε | D,F | Ε | Н | G,I | J |

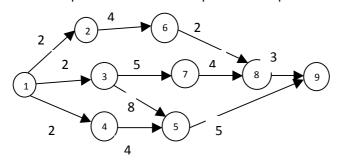
2. A project has the following time Schedule. Construct a PERT network and compute Critical Path and its duration. Also calculate Total float ,Free float.

| Activity | 1-2 | 1-3 | 2-4 | 3-4 | 3-5 | 4-9 | 5-6 |
|---------------|-----|-----|-----|-----|------|------|-----|
| Time in Weeks | 4 | 1 | 1 | 1 | 6 | 5 | 4 |
| Activity | 5-7 | 6-8 | 7-8 | 8-9 | 8-10 | 9-10 | |
| Time in Weeks | 8 | 1 | 2 | 1 | 8 | 7 | |

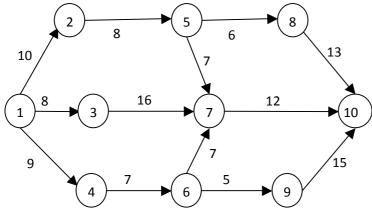
3. A project schedule has the following characteristics. Construct the PERT network and find the critical path and time duration of the project.

| Activity | 1 - 2 | 1 - 4 | 1 - 7 | 2 - 3 | 3 - 6 | 4 - 5 | 4 - 8 | 5 - 6 | 6 - 9 | 7 - 8 | 8 - 9 |
|----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| Time | 2 | 2 | 1 | 4 | 1 | 5 | 8 | 4 | 3 | 3 | 5 |

4. Write down the procedure to obtain optimum completion time using Critical Path method.



5. Find the critical path and calculate the Total float and Free float for the following PERT diagram.



6. A small maintenance project consists of the following 12 jobs

| Jobs | Duration | Jobs | Duration | Jobs | Duration |
|------|----------|------|----------|------|----------|
| 1003 | in days | 1002 | in days | 1003 | in days |
| 1-2 | 2 | 3-5 | 5 | 6-10 | 4 |
| 2-3 | 7 | 4-6 | 3 | 7-9 | 4 |
| 2-4 | 3 | 5-8 | 5 | 8-9 | 1 |
| 3-4 | 3 | 6-7 | 8 | 9-10 | 7 |

Draw the arrow network of the project. Determine the critical path.

7. A project has the following time schedule:

| Activity | Time | Activity | Time | Activity | Time |
|----------|----------|----------|----------|----------|----------|
| | In month | | In month | | In month |
| 1-2 | 2 | 3-6 | 8 | 6-9 | 5 |
| 1-3 | 2 | 3-7 | 5 | 7-8 | 4 |
| 1-4 | 1 | 4-6 | 3 | 7-9 | 3 |
| 2-5 | 4 | 5-8 | 1 | | |

Construct PERT network and compute total float for each activity.

Find Critical path with its duration.

8. Installaion of air population control equipments requires following activites: Draw network diagram from the following activites and find Citical Path

| Activity | Α | В | С | D | Ε | F | G | Н |
|-----------------------|------|------|---|---|---|---|-----|-----|
| Time | 2 | 3 | 2 | 4 | 4 | 3 | 5 | 2 |
| Immediate Predecessor | none | none | Α | В | С | С | D,E | F,G |

9. Draw network diagram from following details and calculate critical path from it.

| Activity | Α | В | С | D | Е | F | G | Н | I | J |
|-------------|----|----|---|---|-----|-----|---|----|-----|-------|
| Time | 15 | 15 | 3 | 5 | 8 | 12 | 1 | 14 | 3 | 14 |
| Immediate | - | - | Α | Α | В,С | В,С | Ε | Ε | D,G | F,H,I |
| Predecessor | | | | | | | | | | |

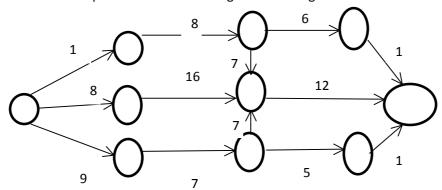
10. Draw network diagram from following details.

| Activity | Α | В | С | D | Ε | F | G | Н |
|-------------|---|---|---|---|-----|---|-----|---|
| Predecessor | - | Α | Α | В | В,С | Ε | D,F | G |
| Activity | | | | | | | | |

11. Listed in the table are the activites and sequencing necessary for a maintenance jobs on the heat exchangers in a refinery.

| Activity | Α | В | С | D | Ε | F | G | Н | 1 | J |
|-------------|---|---|---|---|---|---|---|-----|-------|---|
| Predecessor | - | Α | В | В | В | С | С | F,G | D,E,H | - |
| Activity | | | | | | | | | | |

12. Determine the critical path from the following network diagram.



13. A project consists of a series or tasks labeled A, B, H, I with the following relationships (W< X, Y, means X and Y cannot start until W is completed; X,Y<W means W cannot start until both X and Y are completed). With this notation, construct the network diagram having the following constraints: A < D,E; B,D < F; C < G; B < H; F,G < I

| | , - | | | | | | | | |
|------|-----|---|----|----|----|----|----|---|----|
| Task | Α | В | С | D | Ε | F | G | Н | I |
| Time | 23 | 8 | 20 | 16 | 24 | 18 | 19 | 4 | 10 |

14. Draw a network diagram for following activity:

| Activity | Description | Required Predecessor | Duration |
|----------|---------------------|----------------------|----------|
| Α | Product design | (None) | 5 months |
| В | Market research | (None) | 1 |
| С | Production analysis | Α | 2 |
| D | Product model | Α | 3 |
| E | Sales brochure | Α | 2 |
| F | Cost analysis | С | 3 |
| G | Product testing | D | 4 |
| Н | Sales training | B, E | 2 |
| I | Pricing | Н | 1 |
| J | Project report | F, G, I | 1 |

Q: 3 Multiple Choice Questions.

- **1.** A dummy activity is required when
 - **a.** Two or more activities have the same starting events.
 - **b.** Two or more activities have different ending events.
 - **c.** Two or more activities have the same ending events.
 - **d.** The network contains two or more activities that have identical starting and ending events.
- 2. Which of the following is a basic assumption of PERT?
 - **a.** There is only one complete route from the start of a project to the end of a project.
 - **b.** Only critical path activities in the network must be performed.
 - c. Activity completion times are known with certainty.
 - **d.** No activity in the network must be repeated.
- 3. PERT analysis computes the variance of the total project completion time as
 - **a.** The sum of the variances of all activities in the project.
 - **b.** The sum of the variances of all activities not on the critical path.
 - c. The variance of the final activity of the project.
 - **d.** The sum of the variances of all activities on the critical path.
- 4. The critical path of a network is the
 - **a.** Path with the fewest activities.
 - **b.** Shortest time path through the network.
 - **c.** Longest time path through the network.
 - d. Path with the most activities.
- 5. Dummy activities
 - a. are found in both AOA and AON networks.
 - **b.** are used when two activities have identical starting and ending events.
 - **c.** have a duration equal to the shortest non-dummy activity in the network.
 - **d.** cannot be on the critical path.
- **6.** Which of the following is a limitation of PERT and CPM?
 - a. The graphical nature of a network delays comprehension of the activity list's interrelationships.
 - **b.** They are applicable to only a narrow variety of projects and industries.
 - **c.** They can be used only to monitor schedules.

- **d.** There is inherent danger of too much emphasis being placed on the critical path.
- 7. The EF of an activity is the
 - **a.** Max{EF of all immediate predecessors}.
 - **b.** Min{LS of all immediate following activities}.
 - **c.** ES + Activity time.
 - **d.** LF-Activity time.
- 8. In a CPM/PERT network a dummy activity is necessary when
 - a. two activities have the same starting node
 - **b.** two activities have the same ending node
 - c. a node does not actually connect to another node
 - **d.** when two activities share the same starting and ending node
- **9.** Shared slack in an activity network is defined as
 - a. The amount of time an activity can be delayed without delaying the entire project.
 - **b.** The amount of slack that an activity has in common with another activity.
 - c. The amount of unused resources for an activity.
 - d. The amount by which a time estimate can be in error without affecting the critical path computations.
- 10. If an activity has zero activity slack it
 - **a.** means that the project is expected to be delayed.
 - **b.** must be a dummy activity.
 - c. is on the critical path.
 - **d.** all of the above
- 11. Which of these is not correct
 - **a.** PERT is probabilistic in nature.
 - **b.** CPM is deterministic in nature.
 - c. CPM is event-oriented.
 - **d.** CPM and PERT use similar terminology but were developed independently.
- **12.** Which of the following is not a rule of network construction?
 - **a.** Each defined activity is represented by one and only one arrow.
 - **b.** A network should have only initial and one terminal node.
 - **c.** Identical initial and final nodes can identify two activities.
 - **d.** Only as few dummy activities should be included as is warranted.
- 13. Which of the following is not correct in respect of PERT calculations?
 - a. Expected time of an activity is a weighted average of three times estimates, a, m, and b with respective weights of 1, 4, and 1.
 - **b.** The completion time of an activity is assumed to follow normal distribution.
 - c. The standard deviation of activity completion time is one-sixth of the excess of pessimistic over optimistic time.
 - **d.** The sum total of variances of critical activity times gives the variance of the overall project completion time.
- **14.** Mark the wrong statement.
 - a. All activities on a critical path are critical activities.
 - **b.** A project network may have none, one, or more critical paths.
 - **c.** A delay in critical activity surely delays the completion of project.
 - **d.** Each critical activity has identical earliest and the latest start times.
- **15.** CPM is:
 - a. Critical Project Management

- **b.** Critical Path Management
- c. Critical Path Method
- d. Crash Project Method
- **16.** Pick the wrong relationship:
 - **a.** Interfering float = Total float Free float
 - **b.** Total float =Free float + Independent float
 - c. Total float ≥ Free float ≥ Independent float
 - **d.** Free float = Total float Head event slack
- **17.** The critical path is the _____ path through the network.
 - a. longest
 - **b.** shortest
 - c. straightest
 - d. none of the above
- **18.** An activity cannot be begun until
 - a. all activities previously begun have been completed
 - **b.** its predecessors have been completed
 - c. its latest start time have completed
 - **d.** all of the above
- 19. Arcs in a project network indicate
 - a. completion times.
 - **b.** precedence relationships.
 - c. activities.
 - **d.** the critical path.
- **20.** The critical path
 - **a.** is any path that goes from the starting node to the completion node.
 - **b.** is a combination of all paths.
 - **c.** is the shortest path.
 - **d.** is the longest path.
- 21. Activities following a node
 - a. can begin as soon as any activity preceding the node has been completed.
 - **b.** have an earliest start time equal to the largest of the earliest finish times for all activities entering the node.
 - c. have a latest start time equal to the largest of the earliest finish times for all activities entering the node.
 - d. None of the alternatives is correct.
- 22. Slack equals

- a. LF EF.
- **b.** EF LF.
- **c.** EF LS.
- **d.** LF ES.
- 23. Activities with zero slack
 - a. can be delayed.
 - **b.** must be completed first.
 - **c.** lie on a critical path.
 - **d.** have no predecessors.

| Q: 4 | Fill | in | the | b | lan | ks. |
|------|------|----|-----|---|-----|-----|
|------|------|----|-----|---|-----|-----|

| is the time an activity can be delayed without affecting the sta other activity. The process of reducing total time that it takes to complete a project by entire the complete and project by entire that it takes to complete a project by entire that it takes to complete a project by entire that it takes to complete a project by entire that it takes to complete and project by entire that it takes to complete a project by entire that it takes to complete and project by entire that the complete t | |
|--|------------------|
| · | nploying |
| 3. The process of reducing total time that it takes to complete a project by en | nploying |
| | |
| additional resources is called | |
| 4. PERT often assumes that the time estimates follow the probabilit | y distribution. |
| 5. In PERT/CPM, a(n) is a time-consuming job or task. | |
| 6. In an AON network representation, the represents the activities. | |
| 7 assumes that activity times are probabilistic, while assum | es that |
| activity times are deterministic. | |
| 8. The earliest start (ES) of an activity is the earliest finish (EF) of | the immediate |
| · | |
| 9. In CPM, we assume that resources are expended in a(n) | |
| 10. Major events in a projects are often referred to as | |
| 11. If we have a project that we have not done before, then is probably to be a project that we have not done before, then is probably to be a project that we have not done before, then is probably to be a project that we have not done before, then is probably to be a project that we have not done before, then is probably to be a project that we have not done before, then is probably to be a project that we have not done before, then is probably to be a project that we have not done before, then is probably to be a project that we have not done before it is probably to be a project that we have not done before it is probably to be a project that we have not done before it is probably to be a project that we have not done before it is probably to be a project that we have not done before it is probably to be a project that we have not done before it is project to be a project that the project tha | e preferred |
| over as a project management scheme. | |
| 12. assumes that activity times are probabilistic, while assumes that | : activity times |
| are deterministic. | |
| 13. In PERT/CPM, a(n) is a time-consuming job or task. | |
| 14. PERT stands for Program evaluation and Review | |
| 15. The path with the longest duration in a network is called | |
| 16. PERT is amodel while CPM is a deterministic model. | |
| 17. The critical path represents the time required to complete the project. | |
| 18. CPM stands for | |

Q: 5 True of False.

- 1. PERT differs from CPM in that PERT provides time versus cost tradeoffs.
- 2. PERT uses three activity time values to obtain an estimate of the mean activity time and predictability of duration.
- 3. Activities on the critical path exhibit positive slack times.
- **4.** The expected activity time is the average of an activity's optimistic and pessimistic times
- 5. Dummy activities are found in activity-on-arc (AOA) convention PERT/CPM network models.

- **6.** The dummy activity has an expected time of zero by definition.
- **7.** Early start time (ES) is computed from right to left (backward pass).
- 8. The project-group approach extends little or no formal authority to the
- **9.** project coordinator.
- 10. PERT, but not CPM, has the ability to consider the precedence relationships in a project.
- **11.** The shortest of all paths through the network is the critical path.
- 12. The fundamental difference between PERT and CPM is that PERT uses the beta distribution for crashing projects while CPM uses cost estimates.
- 13. Slack is the amount of time an activity can be delayed without delaying the entire project.
- 14. Every network has at least one critical path.
- 15. The critical path can be determined by use of either the "forward pass" or the "backward pass."
- **16.** The PERT pessimistic time estimate is an estimate of the minimum time an activity will require.
- 17. The standard deviation of project duration is the average of the standard deviation of all activities on the critical path.
- 18. In PERT analysis, the identification of the critical path can be incorrect if a noncritical activity takes substantially more than its expected time.
- 19. PERT stands for Program Efficiency and Review Techniques.
- **20.** There can be multiple critical paths on the same project, all with different durations.