Semester – IV : 060090407 – GE4 Application of Algebra

Question Bank

Unit-1	Balanced Incomplete Block Designs (BIBD)
[A]	**1 – Mark Questions**
1.	Define BIBD.
2.	Define a difference set.
3.	Define Incidence Matrix of BIBD
4.	Define Quadratic residue modulo.
5.	Define Difference set families.
6.	Define a difference set.
7.	Show that there is no (v, b, r, k, λ) – BIBD with v = 10, k = 7, $\lambda = 1$.
[B]	**3 – Marks Questions**
1.	An Advertising agency wishes to test five brands of a beauty lotion on 10 subjects. Each of the subjects is given three brands to try. Find a scheme for allocating the brands to subjects such that each brand is tried by the same number of subjects and every two brands are tried together by the same number of subjects. How many subjects try any given brand? How many subjects try any given pairs of brands?
2.	Construct a symmetric (7, 4, 2)-design and obtain the derived BIBD.
3.	Find a 2-fold difference set family in the additive group Z13. What are the parameters of the BIBD induced by it?
4.	Let V = {1, 2, 3, 4, 5, 6, 7} and let D = {B1,B2, B3, B4, B5, B6, B7}, where B1 = {1, 2, 4} B2 = {2, 3, 5}, B3 = {3,4, 6}, B4 = {4, 5, 7}, B5 = {5, 6, 1}, B6 = {6, 7, 2}, B7 = {7, 1, 3}. Show that D is symmetric BIBD, and find its complement D'. What are the parameter of D'?
5.	Find the BIBDs determined by the difference set of S = {1, 2, 4} for G = Z7 be the additive group of integers modulo 7.
6.	Find the set of quadratic residues modulo 11, and construct the symmetric BIBD determined it.
7.	Prove that If A be the incidence matrix of a BIBD, then AAT is a non-singular matrix.
8.	Construct a symmetric (7, 4, 2)-design and obtain the derived BIBD.
9.	Construct the BIBD, D induced by the differences set family of G = Z9, S1 = {0, 1, 2, 4} and S2 = {0, 3, 4, 7}.
10.	A number of volleyball teams of five students each are formed from a group of 15 students in such a way that every student plays on the same number of teams, and any pair of students play together on exactly two teams. How many teams are there?
11.	Find a 3-fold difference set family in the additive group Z13. What are the parameters of the BIBD induced by it?
12.	Construct a BIBD with parameter (7, 14, 6, 3, 2).
[C]	**5 – Marks Questions**
1.	A schoolmistress takes 15 girls out for a daily walk, with the girls arranged in rows of three each. Design an arrangement of rows for seven consecutive days so that any two girls walk together in the same row exactly once.
2.	State and Prove that Fisher's inequality.
3.	Prove that if p be a prime number greater than 2, then there are (p -1)/2 quadratic residues modulo p and Qp = {resp(n2) | 1 ≤ n ≤ (p -1)/2}.

4.	Construct a BIBD on the set {1, 2, 3, 4, 5, 6} with parameters (6, 10, 5, 3, 2).
5.	Let V be a set of v elements, and let D = V(k) be the set of all subsets of V having k elem k < v. Then D is a balanced incomplete block design on V with parameters (v, b, r, k, λ) where b = $\binom{v}{k}$, r = $\binom{v-1}{k-1}$, λ = $\binom{v-2}{k-2}$. In particular, if k = v - 1, the D is a symmetric BIBD with parameter (v, v, v – 1, v – 1, v – 2).
6.	Prove that if F be a finite field of order 6t + 1, and let a be a primitive element in F. Let Si = {ai, a2t+i, a4t+i}, i = 0,1,….,t – 1
7.	Let G = Z9, S1 = {0, 1, 2, 4} and S2 = {0, 3, 4, 7}. Show that S1, S2 form a difference set family and find its parameters.
8.	Construct a symmetric BIBD with parameters (13, 4, 1).Hence obtain a BIBD with parameters (9,12,8,6,5).
9.	Prove that if D = {B1,…..,Bv} be a symmetric BIBD on the set V = { a1,…..,av } with param k , λ). Let B*1,…..B*v be subsets of V given by ai ∈ B*j ⇔ aj ∈ Bi i.j = 1,…,v Then D* = { B*1,…..B*v } is a symmetric BIBD with parameters (v, k , λ).
10.	Construct a 3-fold difference set family in Z19.
11.	Prove that if F be a finite field of order 4t + 1, and let a be a primitive element in F. Let Si = {ai, at+i, a2t + i, a3t+i}, i = 0,1,….,t – 1
12.	Verify that {0,1,2,5,12, 18,22,24,26,27,29,32,33} is a difference set in Z40.What is its parameters?
13.	Find a difference set in Z31.
14.	Prove that If S = {s1,…..,sk} be a difference set in group G = {g1,….,gv} with parameters (For each i = 1,…,v, let Bi = gi + S = {gi + S| s ∈ S}, then D ={B1,….,Bv} is a symmetric BIBD set G with parameters (v, k, λ).
Unit-2	**Coding Theory**
[A]	**1 – Mark Questions**
1.	Define Hamming code.
2.	Define Syndrome.
3.	Define parity-check matrix of code C.
4.	Define Hamming code.
5.	Define Canonical generator matrix.
6.	Define q-ary Hamming code.
7.	If P = 0.01 then Find Probability of error and correction in code.
8.	Define Generator polynomial of Cyclic code C.
9.	Define linear code.
10.	Define a perfect code.
11.	Define Generator matrix of the code C.
[B]	**3 – Marks Questions**
1.	Explain the concept of error–correcting code with example.
2.	Find a generator matrix and a parity–check matrix of the binary code C = {000,111}.
3.	Find all binary cyclic codes of length 6.
4.	Find the dual of the binary code C = {0000,1111}, and show that C is a self-orthogonal.
5.	Show that the code C with generator matrix G = $\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$ is cyclic.
6.	Find the word error probability of the code C for length of code words 3

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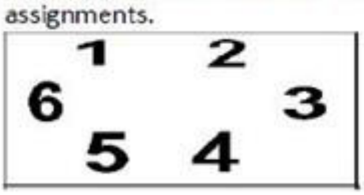
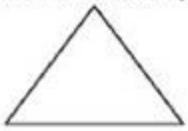
7.	Find all binary cyclic codes of length 3.
8.	Show that a binary (23, 4096, 7)-code if it exists is perfect.
9.	Find all binary cyclic codes of length 5.
10.	Find all binary cyclic codes of length 4.
11.	Find the canonical generator matrix of the code C of H = $\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix}$.
[C]	**5 – Marks Questions**
1.	Prove that if H be a parity-check matrix of an [n, k]-code C over F. then d(c) is equal to the minimal number of linearly dependent columns in H. Consequently , $d(c) \le n - k + 1$.
2.	Prove that Ham(r, q) is a perfect code with minimum distance 3.
3.	Prove that the binary [23, 12, 7] Golay code and the ternary [11, 6, 5] Golay code are both perfect.
4.	Explain the concept of error–correcting code with example.
5.	Write the Syndrome table for the code G = $\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$. Decode the vector 11010.
6.	Write a generator matrix and parity-check matrix for a cyclic Hamming code Ham(3, 2).Obtain the complete code.
7.	Prove that if C be an [n, k]- code. Let G and H be respectively, a generator matrix and a parity-check matrix of C. Then $GH^T = 0 = HG^T$.
8.	Prove that Ham(r, q) is a perfect code with minimum distance 3.
9.	Find the binary linear code C with parity-check matrix H = $\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix}$ and write a generator matrix G of C. Also find the dual code $C^{\perp}$.
10.	Prove that If C be a nonzero ideal in the ring $F[x]_n$. Then (a) There exists a unique monic polynomial g(x) of least degree in C. (b) g(x) divides $x^n - 1$ in F[x]. (c) for all $a(x) \in C$, g(x) divides a(x) in F[x]. (d) C = {g(x)}.
11.	Prove that the binary [23, 12, 7] Golay code and the ternary [11, 6, 5] Golay code are both perfect.
12.	Prove that if C be a code with minimum distance d. Let $t = \left\lfloor \frac{d-1}{2} \right\rfloor$. Then a) C can detect upto d – 1 errors in any transmitted codeword. b) C can correct upto t errors in any transmitted codeword.
13.	Write a standard array for the binary code with the generator matrix G = $\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$ and decode the received vector 0111.
14.	Show that the word error probability of the binary repetition code of length 5 is $(10 - 15p + 6p^2)p^3$, where p is the symbol error probability of the channel.
15.	Prove that if p(x) be a primitive irreducible polynomial of degree r over $F = F^2$. Let $n = 2^r - 1$.Then the cyclic code with generator polynomial p(x) in the ring $F[x]_n$ is Ham(r, 2).
Unit-3	**Symmetry Groups and Colour Patterns**
[A]	**1 – Mark Questions**
1.	Define Dihedral group.

2.	Define Conjugate.
3.	Define symmetry.
4.	Define transposition.
5.	Define cycle of length.
6.	Find the cycle of the length for $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 2 & 1 & 3 & 5 \end{pmatrix}$.
7.	Define action of G.
[B]	**3 – Marks Questions**
1.	Find the number of patterns obtained on coloring the vertices of a square with m colors.
2.	Find the symmetry groups of the conic sections ellipse, parabola and hyperbola.
3.	Find the cycle index polynomial of S4.
4.	Find the cycle index polynomial of S3.
5.	Find the cycle index polynomial of S5.
6.	Show that the permutations $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 7 & 5 & 1 & 6 & 4 & 2 & 3 \end{pmatrix}$ $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 4 & 1 & 6 & 2 & 3 & 7 & 5 \end{pmatrix}$ have the same cyclic structure. Find σ = such that $\beta = \sigma \alpha \sigma^{-1}$.
7.	Find the symmetry groups of the letters of the alphabet: F, L, E, Y, H, I.
8.	Find the number of patterns obtained on coloring the vertices of a rectangle with m colors.
9.	Find the cycle decomposition of the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 \\ 4 & 3 & 8 & 7 & 1 & 6 & 5 & 2 & 16 & 9 & 10 & 15 & 11 & 14 & 12 & 13 \end{pmatrix}$.
[C]	**5 – Marks Questions**
1.	Explain colorings and coloring patterns for equilateral and isosceles triangle.
2.	State and prove Burnside theorem.
3.	A rectangular dining table seats six persons, two along each longer side and one on each shorter side. A colored napkin having one of m given colors is placed for each person. Find the number of distinct patterns among all possible color assignments. 
4.	Find the number of nonisomorphic graphs on six vertices
5.	Prove that α, β be permutations of a finite set X. Then α, β have the same cyclic structure if and only if α, β are conjugate elements in the group S_x.
6.	Write the classic structure classification table for S4.
7.	Write the classic structure classification table for S6.
8.	A straight necktie in the form of a long rectangular strip is divided into n bands of equal width parallel to the shorter side. Each band is colored by one of m given colors. Find the number of toes with distinct patterns. [1|2| | | | | | n].
9.	Find the generating function $f_4(x)$ for the nonisomorphic graphs on four vertices.

10.	Prove that if X be a finite set. Let $\sigma \in S_x$, $\sigma \neq e$. Then σ can be expressed as a product of pairwise disjoint cycles of lengths greater than 1.This decomposition is unique, except for the order in which the cycles are written.
11.	Each vertex of an equilateral triangle is colored by one of m given colors. Find the number of distinct patterns among all possible colorings. 
Unit-4	**Special Types of Matrices**
[A]	**8 – Mark Questions**
1.	Discuss all the types of special matrices with example.
2.	State and prove Frobenius- König theorem
3.	Prove that if n > 2. A necessary condition for an n-square matrix A to be a Hadamard matrix is that n is a multiple of 4.
4.	Prove that a permutation matrix is irreducible if and only if it is permutation similar to a primary permutation matrix.
5.	Let A and B ne n-square positive semi definite matrices, then there exists an invertible matrix P such that P*AP and P*BP are both diagonal matrices. In addition, if A is nonsingular matrix, then P can be chosen so that P*AP =I and P*BP is diagonal.
[B]	**7 – Marks Questions**
1.	1. Prove that $A \in M_n$ be a positive definite matrix and let B be an n X m matrix. Then for any positive semi definite $X \in M_m$, $\begin{pmatrix} A & B \\ B^* & X \end{pmatrix} \geq 0 \Leftrightarrow X \geq B^* A^{-1} B$. 2. Let A be an kn X kn positive semi definite matrix partitioned as $A = (A_{ij})$, where each A_{ij} is an n X n matrix, $1 \leq i,j \leq k$. Then the k X k matrix $D = (\det A_{ij})$ is positive semi definite.
2.	Let A and B ne n-square positive semi definite matrices, then there [7] exists an invertible matrix P such that P*AP and P*BP are both diagonal matrices. In addition, if A is nonsingular matrix, then P can be chosen so that P*AP =I and P*BP is diagonal.
3.	Let $A \geq 0$ and $B \geq 0$ be of the same size. Then 1. The trace of the product AB is less than or equal to the product of the traces tr A and tr B; that is, $tr(AB) \leq trA\, trB$. 2. The eigenvalues of AB are all nonnegative. Furthermore, AB is positive semi definite if and only if AB = BA. 3. If α, β are the largest eigenvalues of A,B respectively, then $\frac{-1}{4}\alpha\beta I \leq AB + BA \leq 2\alpha\beta I$.
4.	State and prove Birkhoff theorem.
5.	Let A and B be positive semi definite matrices. Then $A \geq B \Rightarrow A^{\frac{1}{2}} \geq B^{\frac{1}{2}}$.