Semester - IV 060090404 - SEC2 Combinatorial Mathematics

Question Bank

Unit-1	Permutation and Combination
[A]	**3 – Marks Questions**
1.	How many numbers greater than 2000 can be formed with the digits 1, 2, 3, 4, 5?
2.	How many numbers greater than 10000 can be formed with the digits 1, 2, 3, 4, 5?
3.	How many numbers greater than 23000 can be formed with 1, 2, 3, 4, 5?
4.	Find the number of ways in which 6 beads can be arranged to form a necklace.
5.	Find the number of ways the letters of the word TRIANGLE to be arranged so that the word ANGLE will be always present.
6.	In how many ways can the word PENCIL be arranged so that N is always next to E.
7.	In how many ways can the letter of word LAUGHTER be arranged so that the vowel may never be separated?
8.	In a ration shop queue 2 boys 2 girls and 2 men are standing in such a way that the boys the girls and the men are together each. Find the total number of ways of arranging the queue.
9.	Which permutation of {1,2,3,4,5} follows 13542 and precedes 34215?
10.	A family of 4 brothers and 3 sisters is to be arranged for photograph in one row. In how many ways can they be seated if no two sisters sit together?
11.	In how many ways can 4 Americans and 4 English men be seated at round table so that no two Americans may be together?
12.	In how many ways can 7 persons be arranged at round table so that 2 particular persons may be together?
13.	Find the number of ways in which a cricket team, consisting of 11 player can be selected from 12 players. Also find how many of these ways include captain.
14.	A boy has 3 library tickets and 8 books of his interest in the library. Of those 8, he does not want to borrow mathematics part II unless Mathematics part I is also borrowed. In how many ways can he choose the three books to be borrowed?
15.	From a class of 14 boys and 10 girls, ten students are to be chosen for competition, at least including 4 boys and 4 girls. The 2 girls who won the prize last year should be included. In how many ways can the selection be made?
16.	A letter lock contains 5 rings each marked with four different letters. Find the number of all possible unsuccessful attempts to open the lock.
17.	How many numbers can be formed with the digits 1, 2, 3, 4, 3, 2, 1 so that odd digits always occupy the odd places?
18.	Determine the mobiles integers in the following permutation. (i) $\overleftarrow{4}\ \overrightarrow{3}\ \overrightarrow{5}\ \overleftarrow{1}\ \overleftarrow{6}\ \overrightarrow{2}\ \overleftarrow{7}\ \overleftarrow{8}$ (ii) $\overrightarrow{8}\ \overleftarrow{2}\ \overrightarrow{1}\ \overrightarrow{5}\ \overleftarrow{6}\ \overrightarrow{7}\ \overrightarrow{3}\ \overleftarrow{4}$ (iii) $\overrightarrow{6}\ \overleftarrow{2}\ \overrightarrow{7}\ \overrightarrow{4}\ \overleftarrow{8}\ \overrightarrow{1}\ \overleftarrow{3}\ \overrightarrow{5}$
19.	Generate the all possible permutation of {1,2,3,4} starting with $\overleftarrow{1}\ \overleftarrow{2}\ \overleftarrow{3}\ \overleftarrow{4}$.
20.	If p(n,4)=24024 then find the value of n.
21.	In C(n,3) = C(n,7) then find the value of n.

22.	Find the number of diagonals that can be drawn by joining the angular points of heptagon.
23.	A computer has 5 terminals and each terminal is capable of four distinct position including the position of rest. What is the total number of signals that can be made?
24.	Determine the inversion of the following permutations of {1,2,3,4,5,6,7,8}: (i) 46721835 (ii) 84125376 (iii) 76328145
25.	Construct the permutations of {1,2,3,4,5,6,7,8} whose inversion sequences are (i) 2,4,5,0,1,3,1,1 (ii) 6,6,1,1,4,3,0,0 (iii) 5,3,2,1,1,0,1,2
26.	Let S={x_7, x_6, ... x_0}. Determine the combinations of S corresponding to the following 8-tuples. (a) 00011011 (b) 01010101 (c) 00001111
27.	How many positive integers of not more than 4 digits can be formed by using the digits 2, 3, 4, 5, 0 any number of times?
28.	How many permutations of {1,2,3,4,5,6} have (i) Exactly 15 inversions? (ii) Exactly 14 inversions? (iii) Exactly 13 inversions?
29.	Bring the permutations 2561347, 1357264 and 2473561 to 123456 by successive switches of adjacent numbers.
Unit-2	**Binomial Coefficients**
[A]	**3 – Marks Questions**
1.	Prove that for all integers n and k with 1≤ k ≤ n-1 $\binom{n}{k}$ - $\binom{n-1}{k}$ = $\binom{n-1}{k-1}$.
2.	Using Binomial theorem prove the following properties: (i) $\binom{n}{0}+\binom{n}{1}+\binom{n}{2}$ +--------+ $\binom{n}{n} = 2^n, \forall n \in N$ (ii) $1\binom{n}{1}+2\binom{n}{2}$ +--------+ $n\binom{n}{n}$ =n 2^{n-1}, where $n \geq 1$
3.	Let S be the set of n elements then prove that clutter on S contains at the most $\binom{n}{\lfloor n/2 \rfloor}$ sets.
4.	For $n_1, n_2 ....n_t \in Z^+$ prove that $\binom{n}{n1\ \ n2\ \ldots\ldots nt} = \frac{n!}{n_1!\ n_2! \ldots\ n_t!}$ where $n_1 + n_2 + ... + n_t = n$.
5.	State and Prove Binomial Theorem.
6.	Prove that $\sum_{k=0}^{n}\binom{n}{k}^2 = \binom{2n}{n}$ where n≥0.
7.	For any $n, k \in N$ prove that

	$\binom{0}{k}+\binom{1}{k}+\binom{2}{k}$ +--------+ $\binom{n-1}{k}+\binom{n}{k}=\binom{n+1}{k+1}$.
8.	Let n be any nonnegative number then prove that $(x_1+x_2+\ldots..+x_n)^n = \sum\binom{n}{n_1\, n_2 \ldots n_t} x_1^{n_1} x_2^{n_2} \ldots. x_t^{n_t}$.
9.	Let n be a positive integer then for all x and y prove that $(x+y)^n = \sum_{k=0}^{n}\binom{n}{k} x^{n-k} y^k$.
10.	For any $r, k \in N$ prove that $\binom{r}{0}+\binom{r+1}{1}+\binom{r+2}{2}$ +--------+ $\binom{r+k}{k}=\binom{r+k+1}{k}$.
11.	State and Prove Multinomial Theorem.
12.	Using binomial theorem prove that $3^n = \sum_{k=0}^{n}\binom{n}{k} 2^k$.
13.	State and prove Pascal's Formula.
14.	Let S be the set of n elements then prove that clutter on S contains at the most $\binom{n}{\lfloor n/2 \rfloor}$ sets.
15.	Using binomial theorem prove that $2^n = \sum_{k=0}^{n}(-1)^k\binom{n}{k} 3^{n-k}$.
16.	Let $n \in N$ then prove that the sequence of binomial coefficients $\binom{n}{0}, \binom{n}{1}, \binom{n}{2}$, -------, $\binom{n}{n}$ is unimodal.
17.	For z = x/y prove that $\frac{1}{(1-z)^n} = \sum_{k=0}^{\infty}\binom{n+k+1}{k} z^k$.
Unit-3	**Inclusion-Exclusion Principle and Applications**
[A]	**5 – Marks Questions**
1.	Prove that the number of objects of S that have none of the properties $P_1, P_2 \ldots P_m$ then $\lvert\bar{A}_1 \cap \bar{A}_2 \ldots. \cap \bar{A}_m\rvert$ = $\lvert S\rvert - \sum\lvert A_i\rvert + \sum\lvert A_i \cap A_j\rvert - \sum\lvert A_i \cap A_j \cap A_k\rvert + \ldots + (-1)^m \lvert A_1 \cap A_2 \cap \ldots \cap A_m\rvert$.
2.	Find the number of integers between 1 and 1000, inclusive that are not divisible by 5, 6, and 8.
3.	A bakery sells chocolate, cinnamon and plain doughnuts and at a particular time has 6 chocolate, 6 cinnamon, and 3 plain. If a box contains 12 doughnuts, how many different options are there for a box of doughnuts?
4.	Determine the number of solutions of the equation $x_1 + x_2 + x_3 + x_4 = 14$ in nonnegative integers x_1 , x_2 , x_3 and x_4 not exceeding 8.
5.	Find the number of integers between 1 and 1000, inclusive that are not divisible by 4, 5, and 6.
6.	Determine the number of 10-combinations of the multiset S={∞.a, 4.b, 5.c, 7.d}.
7.	Determine the number of solutions of the equation $x_1 + x_2 + x_3 + x_4 = 20$ that satisfy $1 \le x_1 \le 6$, $0 \le x_2 \le 7$, $4 \le x_3 \le 8$, $2 \le x_4 \le 6$.
8.	Prove that for n≥1 $D_n = n!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\ldots+(-1)^n\frac{1}{n!}\right)$.

9.	Find the number of integers between 1 and 1000, inclusive that are not divisible by 4, 6, 7 and 10.
10.	How many permutations of the letters M,A,T,H,I,S,F,U,N are there such that none of the words MATH, IS and FUN occur as consecutive letters?
11.	Determine the number of permutations of $\{1,2,\ldots 8\}$ in which no even integer is in its natural position.
12.	Prove that the number of ways to place n non-attacking, indistinguishable rooks on n-by-n board with the forbidden positions equals; $n! - r_1(n-1)! + r_2(n-2)! - \ldots.(-1)^k r_k(n-k)! + \ldots + (-1)^n r_n$.
13.	Find the number of integers between 1 and 1000, inclusive that are not divisible by 4, 7, and 9.
14.	How many permutations of the letters M,A,N,I,N,B,L,A,C,K are there such that none of the words MAN, IN and BLACK occur as consecutive letters?
15.	Determine the number of permutations of $\{1,2,\ldots 8\}$ in which exactly four integers are in their natural position.
16.	Prove that for n ≥ 1, $Q_n = n! - \binom{n-1}{1}(n-1)! + \binom{n-1}{2}(n-2)! - \binom{n-1}{3}(n-3)! + \ldots + (-1)^{n-1}\binom{n-1}{n-1}1!$.
17.	Find the number of integers between 1 and 1000 that are neither perfect squares not perfect cubes.
18.	Determine the number of 12-combinations of the multiset S = {4.a, 3.b, 4.c, 5.d}.
19.	Determine the number of permutations of $\{1, 2, \ldots 9\}$ in which at least one odd integer is in its natural position.
Unit-4	**Recurrence Relations and Generating Functions**
[A]	**5 – Marks Questions**
1.	Prove that the Fibonacci number satisfy the formula $f_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n, n \geq 0$.
2.	Solve the differential equation y’’ – 4y’ + 4y = 0.
3.	Solve $h_n = 2h_{n-1} + n^3$, $(n\geq1)$ $h_0 = 0$.
4.	Word of length n using only the three letters a, b, c, are to be transmitted over communication channel subject to the condition that two word in which two a’s appear consecutively is to be transmitted. Determine the number of words allowed by the communication channel.
5.	Prove that if $q_1, q_2, \ldots. q_t$ be the distinct roots of the characteristic equation $h_n - a_1h_{n-1} - a_2h_{n-2} - \ldots - a_kh_{n-k} = 0$, $(a_k \neq 0, n \geq k)$, then, if q_i is an s_i-fold root of the above characteristic equation then part of the general solution of this recurrence relation corresponding to q_i is;

	${H_n}^{(i)} = c_1 {q_i}^n + c_2 {q_i}^n + \cdots + c_{si} n^{si-1} {q_i}^n$. The general solution of recurrence relation is $h_n = {H_n}^{(1)} + {H_n}^{(2)} + \ldots + {H_n}^{(t)}$.
6.	Find the general solution of recurrence relation $h_n - 4h_{n-1} + 4h_{n-2} = 0,\ (n \geq 2)$.
7.	Show that let k be an integer, and let the sequence $h_0, h_1, \ldots, h_n, \ldots$ be defined by letting h_n equal the number of nonnegative integral solution of $e_1 + e_2 + \ldots + e_k = n$.
8.	Solve the recurrence relation $h_n = -h_{n-1} + 3h_{n-2} + 5h_{n-3} + 2h_{n-4},\ (n \geq 4)$ subject to the initial values $h_0 = 1$ and $h_3 = 2$.
9.	Prove that, if q be a nonzero number. Then $h_n = q^n$ is a solution of linear homogeneous recurrence relation $h_n - a_1 h_{n-1} - a_2 h_{n-2} - \ldots - a_k h_{n-k} = 0,\ (a_k \neq 0, n \geq k)$. with constant coefficients if and only if q is a root of the polynomial equation $x^k - a_1 x^{k-1} - a_2 x^{k-2} - \ldots - a_k = 0$. If the polynomial equation has k roots $q_1, q_2, \ldots, q_k$, then $h_n = c_1 {q_1}^n + c_2 {q_2}^n + \ldots + c_k {q_k}^n$ is a general solution.
10.	Solve $h_n = 3h_{n-1} - 4n,\ (n \geq 1)$ $h_0 = 2$.
11.	Solve the differential equation $y'' - 4y' + 4y = 0$.
12.	There are three pegs and n circular disks of increasing size on one peg, with largest disk on the bottom. These disks are to be transferred, one at a time, onto another of the peg, with the provision that at no time is one allowed to place a larger disk on top of a smaller one. The problem is to determine the number of moves necessary for the transfer. *(Tower of Hanoi puzzle)*
13.	Solve $h_n = 3h_{n-1} + 3^n,\ (n \geq 1)$ $h_0 = 2$.
14.	Show that let k be an integer, and let the sequence $h_0, h_1, \ldots, h_n, \ldots$ be defined by letting h_n equal the number of nonnegative integral solution of $e_1 + e_2 + \ldots + e_k = n$.
15.	Prove that if $q_1, q_2, \ldots, q_t$ be the distinct roots of the characteristic equation $h_n - a_1 h_{n-1} - a_2 h_{n-2} - \ldots - a_k h_{n-k} = 0,\ (a_k \neq 0, n \geq k)$, then, if q_i is an s_i-fold root of the above characteristic equation then part of the general solution of this recurrence relation corresponding to q_i is; ${H_n}^{(i)} = c_1 {q_i}^n + c_2 {q_i}^n + \cdots + c_{si} n^{si-1} {q_i}^n$. The general solution of recurrence relation is $h_n = {H_n}^{(1)} + {H_n}^{(2)} + \ldots + {H_n}^{(t)}$.
16.	Solve the recurrence relation $h_n = 5h_{n-1} - 6h_{n-2},\ (n \geq 2)$ $h_0 = 1$ and $h_1 = 1$.
17.	Prove that nth Fibonacci number f_n satisfies $f_n = \binom{n-1}{0} + \binom{n-2}{1} + \binom{n-3}{2} + \ldots + \binom{n-k}{k-1}$.

18.	Determine the generating function for the number of n-combinations of apples, bananas, oranges and pears where, in each n-combination, the number of oranges is between 0 and 4, and there is at least one pear.
19.	Solve $h_n = 2h_{n-1} + 3^n$, $(n\geq 1)$ $h_0 = 2$.
20.	There are three pegs and n circular disks of increasing size on one peg, with largest disk on the bottom. These disks are to be transferred, one at a time, onto another of the peg, with the provision that at no time is one allowed to place a larger disk on top of a smaller one. The problem is to determine the number of moves necessary for the transfer. *(Tower of Hanoi puzzle)*