Semester – IV :	**060090402 - CC9 Higher Order Differential Equations and Transforms**

Question Bank

Unit-1	Series Solutions
[A]	**1 – Mark Questions**
1.	Define Analytic function.
2.	Write the Rodrigue's formula.
3.	Define ordinary point.
4.	Write the definition of Regular singular point.
5.	Define Singular point.
6.	Define Generating function.
7.	Define Irregular singular point.
[B]	**3 – Marks Questions**
1.	Determine the value of $J_{\frac{3}{2}}(x)$.
2.	Using generating function for Legendre's polynomials, show that $P_n(1) = 1$
3.	Using $(1-2xt+t^2)^{-\frac{1}{2}} = \sum_{n=0}^{\infty} P_n(x)t^n$ for Legendre's polynomials, show that $P_n(-1) = (-1)^n$.
4.	Determine the value of $J_{\frac{1}{2}}(x)$.
5.	Using generating function for Legendre's polynomials, show that $P_n(-x) = (-1)^n P_n(x)$.
6.	Find the value of $J_{-\frac{3}{2}}(x)$.
7.	Obtain the value of $J_{-\frac{1}{2}}(x)$.
8.	Classify the singularities for the following differential equation $y^{II} + 3x^2y^I + 2xy = 0$.
9.	Obtain the value of $P_n(0)$ using $(1-2xt+t^2)^{-\frac{1}{2}} = \sum_{n=0}^{\infty} P_n(x)t^n$ formula.
[C]	**5 – Marks Questions**
1.	Express $2 - 3x + 4x^2$ in terms of Legendre's polynomials.
2.	If possible then find the series solution of $y^{II} = y^I$.
3.	Prove that $J_{-n}(x) = (-1)^n J_n(x)$.
4.	Prove that $P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$
5.	Find the ordinary points of the equation $(x^2+1)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + 6y = 0$.
6.	Prove that $2nJ_n(x) = x[J_{n+1}(x)+J_{n-1}(x)]$
7.	Express $x^4 - 2x^3 + 3x^2 - 4x + 5$ in terms of Legendre polynomials, by using Rodrigue's formula.
8.	Solve the equation of $\frac{d^2y}{dx^2} + y = 0$ by the power series method.
9.	Prove that $\frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x)$
10.	Prove that $J_n^I(x) = \frac{1}{2}[J_{n-1}(x) - J_{n+1}(x)]$
11.	State and prove Rodrigue's formula.
12.	By the power series method, Solve the equation of $y^{II} + y = 0$. Where $y^{II} = \frac{d^2y}{dx^2}$
13.	Obtain the series solution of $(1-x^2)y^{II} - 2xy^I + 2y = 0$
14.	Determine that $x = 1$ is a regular singular point of $(1-x^2)y^{II} - 2xy^I + n(n+1)y =$

	0
Unit-2	**Laplace transforms**
[A]	**1 – Mark Questions**
1.	Using the definition of Laplace transform find ; $f(t) = e^{-at}, a$ is constant
2.	Define Laplace transform.
3.	Using the definition of Laplace transform find $f(t) = \sin hat$, $s > \|a\|$
4.	State and prove Linearity property.
5.	Find Laplace transform of $e^{at}t^n$
6.	Using the definition of Laplace transform find $f(t) = \cos at$
7.	Write statement of convolution theorem of Laplace transform.
8.	State and prove first shifting property.
9.	Find Laplace transform of $\cos h3t \cos 2t$.
[B]	**3 – Marks Questions**
1.	State and prove division by t property.
2.	Find the Laplace transform of $\left[\frac{e^{-\alpha t}-e^{-\beta t}}{t}\right]$
3.	If $f(t)$ is continuous and $f^I(t)$ is piecewise continuous in the interval $0 \le t \le T$ for any finite T, and $f(t)$ and $f^I(t)$ are of exponential order as $t \to \infty$ then prove $L\{f^I(t)\} = sL\{f(t)\} - f(0)$.
4.	Find the inverse Laplace transform of $\frac{1}{\sqrt{s+a}}$
5.	If $L\{f(t)\} = F(s)$, then prove $L\{f(at)\} = \frac{1}{a}F\left(\frac{s}{a}\right)$
6.	Find $L\left[\int_0^t \frac{\sin x}{x}\right] dx$
7.	If $L\{f(t)\} = F(s)$ then prove that $L\{tf(t)\} = -\frac{d}{ds}F(s)$ and $L\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n}F(s)$
8.	Find $L\left[\frac{\sin wt}{t}\right]$
9.	Using second shifting property find the inverse Laplace transform of $\frac{e^{-2s}}{s-3}$.
10.	Find $L(2t+3)^2$.
[C]	**5 – Marks Questions**
1.	If $f(t)$ and $g(t)$ are the inverse Laplace transforms of $F(s)$ and $G(s)$ respectively then the inverse Laplace transform of the product $F(s)G(s)$ is $\int_0^t f(u)g(t-u)du$. Symbolically $L^{-1}[F(s)G(s)] = L^{-1}\{F(s)\} * L^{-1}\{G(s)\}$
2.	Find the inverse Laplace transform of $\frac{2s^2-6s+5}{s^3-6s^2+11s-6}$
3.	Find the Laplace transform of (I) $4 \sin \mathrm{h3t} + 3 \cos \mathrm{h5t}$ (II) $3\mathrm{t}^2\mathrm{e}^{-2\mathrm{t}} + 5\mathrm{e}^{3\mathrm{t}}\cos 2\mathrm{t}$
4.	Find the Laplace transform of the following

	(I) $f(t) = 5t^3 + 3t^2 - 6t + 3e^{-5t}$ (II) $L\{e^{-3t}(2cos5t - 3sin5t)\}$
5.	Solve the equation $y^{II} - 3y^I + 2y = 4t + e^{3t}$, when $y(0) = 1, y^I(0) = -1$
6.	Find $L^{-1}\left(\frac{1}{(s^2+a^2)^2}\right)$, using convolution theorem.
7.	Find the inverse Laplace transform of $\frac{5s+3}{(s-1)(s^2+2s+5)}$
8.	Find : (I) $L\{cos^2 3t + \sin 5t \sin 2t\}$ (II) $L\{e^{3t} sin^2 t\}$
9.	Using convolution theorem, find the inverse transforms of $\frac{1}{s(s^2+a^2)}$
10.	Find the Laplace transform of $\{e^t(t^2 - 2t + 4)sint\}$
11.	State and prove convolution theorem.
12.	Using unit step function, find Laplace transform of $f(t) = \begin{cases} \sin t, & 0 \le t < \pi \\ \sin 2t, & \pi \le t < 2\pi \\ \sin 3t, & t \ge 2\pi \end{cases}$
13.	Find the value of the following: (I) $L\left[\frac{e^{-bt}-e^{-at}}{t}\right]$, $(a \ne b)$ (II) $L\left[\frac{\cos at - \cos bt}{t}\right]$
14.	Solve the equation $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = e^{-t}sint$, $x(o) = 0, x^I(0) = 1$
15.	Find the Laplace transform of $\int_0^t \frac{1-e^{-at}}{t} dt$
Unit-3	**Fourier series**
[A]	**5 – Marks Questions**
1.	Obtain Fourier series for $f(x) = e^{-x}$ in the interval $0 < x < 2\pi$
2.	Find Fourier series for the function $f(x)$ given by $f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi \le x \le 0 \\ 1 - \frac{2x}{\pi}, & 0 \le x \le \pi \end{cases}$ Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$
3.	Find the Fourier series of the function. $f(x) = \begin{cases} 0, & -2 < x < -1 \\ k, & -1 < x < 1 \\ 0, & 1 < x < 2 \end{cases}$
4.	State and prove Parseval's theorem on Fourier constants.
5.	Obtain Fourier series to represent $f(x) = x + x^2, -\pi < x < \pi$ and deduce that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$.
6.	Find Fourier series for the function $f(x)$ given by

	$f(x) = \begin{matrix} \pi + 2x, -\pi \le x \le 0 \\ \pi - 2x, 0 \le x \le \pi \end{matrix}$
7.	Find the Fourier series to represent the function $f(x) = 2x - x^2$ in $0 < x < 3$
8.	Find the Fourier sine series for $f(x) = \pi x - x^2$ in $(0, \pi)$.
9.	Obtain Fourier series to represent $f(x) = xsinx$, $0 < x < 2\pi$
10.	Find Fourier series expansion for the function $f(x) = \begin{matrix} 0, -\pi < x \le 0 \\ \sin x, 0 \le x < \pi \end{matrix}$ Hence evaluate $\frac{1}{1\cdot 3} - \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} - \cdots = \frac{\pi - 2}{4}$
11.	Find the Fourier series of $f(x) = x^2 - 2$ in $-2 \le x \le 2$.
12.	If the Fourier series expansion of $f(x)$ over an interval $c < x < c + 2l$ is given by $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left\{a_n cos\frac{n\pi x}{l} + b_n sin\frac{n\pi x}{l}\right\}$ then prove $\int_c^{c+2l}[f(x)]^2 dx = l\left[\frac{a_0^2}{2} + \sum_{n=1}^{\infty}(a_n^2 + b_n^2)\right]$
13.	Find Fourier series expansion of $f(x) = \|x\|$, $-\pi < x < \pi$.
14.	If $f(x) = \begin{matrix} -1 + x, & -\pi < x < 0 \\ 1 + x, & 0 < x < \pi \end{matrix}$ Find the Fourier series of $f(x)$.
15.	Find the Fourier series for $f(x) = x^2$ in $-1 < x < 1$.
16.	Express $\sin x$ as cosine series in $0 < x < \pi$.
17.	Find Fourier series representation of $f(x) = \sqrt{1 + cosx}$ in the interval $-\pi < x < \pi$.
18.	If $f(x) = \begin{matrix} x, 0 \le x \le \pi \\ 2\pi - x, \pi \le x \le 2\pi \end{matrix}$ Find the Fourier series of $f(x)$, Deduce that $\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$
19.	Find Fourier series of $f(x) = e^{-x}$ in $-l < x < l$.
Unit-4	**Fourier Transforms**
[A]	**5 – Marks Questions**
1.	If $f(x)$ is continuous at x, then prove $f(x) = \int_0^{\infty}[A(s)\cos sx + B(s)\sin sx]ds$, Where $A(s) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(u)\cos su\, du$, $B(s) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(u)\sin su\, du$
2.	Using Fourier cosine integral formula is given by $\int_0^{\infty}\frac{cos\lambda x}{\lambda^2+1}d\lambda = \frac{\pi}{2}e^{-x}$, $x \ge 0$.
3.	Find the Fourier sine transform of $e^{-\|x\|}$.
4.	Find Fourier transform of $f(x) = \begin{cases} x, \|x\| \le a \\ 0, \|x\| > a. \end{cases}$
5.	If $f(x)$ is an odd function, then prove $f(x) = \frac{2}{\pi}\int_0^{\infty}\sin sx\left\{\int_0^{\infty} f(u)\sin su\, du\right\}ds$.
6.	Show that $e^{-x}\cos x = \frac{2}{\pi}\int_0^{\infty}\frac{(u^2+2)\cos ux}{u^2+4}du$, $x > 0$.
7.	Find the Fourier cosine transform of e^{-x^2}.
8.	Find the Fourier transform of $F(x) = \begin{cases} x^2, \|x\| < a \\ 0, \|x\| > a. \end{cases}$
9.	If $f(x)$ is an even function, then prove $f(x) = \frac{2}{\pi}\int_0^{\infty}\cos sx\left\{\int_0^{\infty} f(u)\cos su\, du\right\}ds$.

10.	Using Fourier sine integral formula, show that $e^{-ax}-e^{-bx}=\frac{2(b^2-a^2)}{\pi}\int_0^\infty \frac{u\sin ux\, du}{(u^2+a^2)(u^2+b^2)}, a\geq 0, b\geq 0$
11.	Find the Fourier complex transform of $f(x)=\begin{cases} e^{-i\omega x}, a<x<b \\ 0, x<a \text{ or } x>b \end{cases}$
12.	Find the Fourier transform of $f(x)=\begin{cases} 1-x^2 \text{ if } \lvert x\rvert<1 \\ 0 \quad \text{if } \lvert x\rvert>1 \end{cases}$
13.	Prove that complex from of Fourier integral theorem $f(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-isx}\{\int_{-\infty}^{\infty} f(u)e^{isu}du\}ds$
14.	Using Fourier cosine integral formula, show that $e^{-ax}=\frac{2a}{\pi}\int_0^\infty \frac{\cos ux}{u^2+a^2}dx, a>0, x\geq 0.$
15.	Find the Fourier cosine transform of $f(x)=\begin{cases} x \quad \text{for } 0<x<1 \\ 2-x \text{ for } 1<x<2 \\ 0 \quad \text{for } x>2 \end{cases}$
16.	Find the Fourier sine transform of $e^{-\lvert x\rvert}$.Hence evaluate $\int_0^\infty \frac{x\sin mx}{1+x^2}dx$
17.	Show that $e^{-x}-e^{-2x}=\frac{6}{\pi}\int_0^\infty \frac{u\sin ux}{(u^2+1)(u^2+4)}, x\geq 0$
18.	Find the Fourier transform of $f(x)=\begin{cases} 1-x^2 \text{ if } \lvert x\rvert<1 \\ 0 \quad \text{if } \lvert x\rvert>1 \end{cases}$ and use it to evaluate $\int_0^\infty \left(\frac{x\cos x-\sin x}{x^3}\right)\cos\frac{x}{2}dx$