DEPARTMENT OF MATHEMATICS

Semester – II : 060090206 – CC4 Linear Algebra

Question Bank

Unit-1	Vector Spaces
[A]	**1 – Mark Questions**
1.	Define span.
2.	Define row space.
3.	Define column space.
4.	Define basis.
5.	Define linear independence set.
6.	Define subspace.
7.	Define linear dependence set
8.	State the dimension theorem for matrices.
9.	Determine the following is subspace of R^3. $W = \{ (x, y, z) / y = x + z + 1 \}$
10.	Determine which of the following is subspace of R^3. $W = \{ (a, 0, 0) / a \in R \}$
11.	Determine which of the following is subspace of P_3. $W = \{a_0 + a_1 x + a_2 x^2 + a_3 x^3 / a_0 = 0\}$
12.	Determine the following is subspace of R^3. $W = \{ (a, b, c) / a = b = 0 \}$
13.	Check the following vectors in R^3 is linearly dependent or linearly independent (1 ,2 , 1) , (3, 6, 3)
14.	. Check the following set of vectors in R^4 is linearly dependent or linearly independent (1 ,0 , -2, 2) , (5, 0, -10, 10)
15.	Check the following set of vectors in R^4 is linearly dependent or linearly independent (1 ,0 , -2, 3) , (5, 0, -10, 15)
16.	Which of the following set of vectors in R^3 is linearly dependent ? (1 ,2 , 1) , (4, 8, 4)
17.	Which of the following set of vectors in R^3 is linearly dependent ? (1 ,2 , 1) , (15, 30, 15)
18.	Let f = $cos^2 x$ and f = $sin^2 x$. Is the function sin2x lie in the space spanned by f and g ?
19.	Let f = $cos^2 x$ and f = $sin^2 x$. Is the function cos 2x lie in the space spanned by f and g ?
[B]	**5 – Marks Questions**
1.	Prove that a finite set of vectors that containing the zero vector is linearly dependent
2.	Prove that If $v_1, v_2, ..., v_r$ are vectors in vector space V then a] The set W of all linear combination of $v_1, v_2, ..., v_r$ is a subspace of V.

	b] It is the smallest subspace of V which contains $v_1, v_2, \ldots, v_r$.
3.	State and prove the necessary and sufficient condition for a non empty subset of a vector space be a subspace
4.	Prove that a set S of with two or more vectors is linearly dependent if at least one vector can be expressed as a linear combination of remaining vectors of S.
5.	State and prove dimension theorem for matrices.
6.	State and prove rank-nullity theorem for matrices.
7.	Is the following polynomials are linearly dependent? $p_1 = 2 + x + x^2$, $p_2 = x + 2x^2$ and $p_3 = 2 + 2x + 3x^2$
8.	Show that the set of all 2×2 matrices of the form $\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix}$ with addition defined by $\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix} + \begin{bmatrix} c & 1 \\ 1 & d \end{bmatrix} = \begin{bmatrix} a+c & 1 \\ 1 & b+d \end{bmatrix}$ and scalar multiplication defined by $k\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix} = \begin{bmatrix} ka & 1 \\ 1 & kb \end{bmatrix}$ is a vector space.
9.	Find a basis for the null space of A, where A = $\begin{bmatrix} 1 & -1 & 3 \\ 5 & -4 & -4 \\ 7 & -6 & 2 \end{bmatrix}$.
10.	Determine whether the set V of all pairs of real numbers (x, y) with the operations $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2 + 1, y_1 + y_2 + 1)$ and $k(x, y) = (kx, ky)$ is a vector space.
11.	Express the polynomial $7 + 8x + 9x^2$ as a linear combinations of $p_1 = 2 + x + 4x^2$, $p_2 = 1 - x + 3x^2$ and $p_3 = 3 + 2x + 5x^2$
12.	Determine b is in the column space of A if so express as linear combination of A. where $A = \begin{bmatrix} 1 & -1 & 1 \\ 9 & 3 & 1 \\ 1 & 1 & 1 \end{bmatrix}, b = \begin{bmatrix} 5 \\ -1 \\ 1 \end{bmatrix}$
13.	Express vector $v = (6, 11, 6)$ as linear combinations of $v_1 = (2, 1, 4), v_2 = (1, -1, 3)$ and $v_3 = (3, 2, 5)$
14.	Find the rank and nullity of $A = \begin{bmatrix} 2 & 0 & -1 \\ 4 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix}$.

15.	Determine whether the following matrices span M_{22} $A_1=\begin{bmatrix}1&0\\0&0\end{bmatrix} A_2=\begin{bmatrix}1&1\\0&0\end{bmatrix} A_3=\begin{bmatrix}1&1\\1&0\end{bmatrix} A_4=\begin{bmatrix}1&1\\1&1\end{bmatrix}$
16.	Determine the dimension of and a basis for the solution space of the system. $x_1 - 4x_2 + 3x_3 - x_4 = 0$ $2x_1 - 8x_2 + 6x_3 - 2x_4 = 0$
17.	Determine the set of all triples of real numbers (x, y, z) with the operations $(x, y, z)+(x', y', z')=(x+x', y+y', z+z')$ and $k(x, y, z)=(kx, y, z)$ is a vector space.
18.	Express vector $v = (1,-2, 5)$ as linear combinations of vectors $v_1 = (1, 1, 1), v_2 = (1, 2, 3)$ and $v_3 = (2, -1, 1)$
19.	Find a basis for the row space of A, where A = $\begin{bmatrix}1&-1&3\\5&-4&-4\\7&-6&2\end{bmatrix}$.
20.	Determine the set of all triples of real numbers (x, y, z) with the operations $(x, y, z)+(x', y', z')=(x+x', y+y', z+z')$ and $k(x, y, z)=(0,0,0)$ is a vector space.
21.	Find rank and nullity of A, where A = $\begin{bmatrix}1&-1&3\\5&-4&-4\\7&-6&2\end{bmatrix}$.
22.	Determine the dimension of and a basis for the solution space of the system. $x_1 + x_2 - 2x_3 = 0$ $-2x_1 - 2x_2 + 4x_3 = 0$ $-x_1 - x_2 + 2x_3 = 0$
Unit-2	**Linear Transformations**
[A]	**1 – Mark Questions**
1.	Define linear transformation.
2.	Define zero transformation.
3.	Define identity transformation.
4.	Define rank of linear transformation.
5.	Define range of linear transformation
6.	Define one one transformation.
7.	Define on to transformation.

8.	State rank –nullity of linear transformation.
9.	Give one example of on -to transformation
10.	Give one example of one one transformation
11.	Give one example of bijective transformation
12.	Give one example of Surjective transformation
[B]	**3 – Marks Questions**
1.	Determine whether the function is a linear transformation. Justify your answer. $T: R^2 \to R^2$, where $T(x,y)=(x+2y,3x-y)$
2.	Determine whether the function is a linear transformation. Justify your answer. $T:P_2 \to P_2$, where $T(p(x)) = (a_0+1)+(a_1+1)x+(a_2+1)x^2$
3.	Determine whether the function is a linear transformation. Justify your answer. $T:R^3 \to R^2, T(x,y,z)=(2x-y+z,y-4z)$
4.	Find domain and codomain of $T_2 \circ T_1$ and find $(T_2 \circ T_1)(x,\ y)$. where $T_1(x,\ y)=(2x,3y)$ and $T_2(x,\ y,)=(x-y,x+y)$.
5.	Find domain and codomain of $T_2 \circ T_1$ and find $(T_2 \circ T_1)(x,\ y)$. where $T_1(x,\ y)=(x-y,y+z,x-z)$ and $T_2(x,\ y,)=(0,x+y+z)$.
6.	Find domain and codomain of $T_2 \circ T_1$ and find $(T_2 \circ T_1)(x,\ y)$. where $T_1(x,\ y)=(x-3y,0)$ and $T_2(x,\ y,)=(4x-5y,3x-6y)$
7.	Determine whether the linear transformation is one-to-one, onto, or both or neither. T: $R^3 \to R^3$, where T (x, y, z) = (x + 3y, y, z +2x)
8.	Determine whether multiplication by A is one-to-one, onto, or both or neither. Where A = $\begin{bmatrix} 1 & 2 \\ 4 & -2 \\ 5 & 3 \end{bmatrix}$
9.	Let $T_1 : R^2 \to R^2$ and $T_2 : R^2 \to R^2$ be the linear operator given by the formula T_1 (x, y) = (x + y, x - y) and T_2 (x, y) = (2x + y, x - 2y) Find the formulas for T_1^{-1} (x, y) and T_2^{-1}(x, y)
10.	Determine whether the linear transformation is one-to-one, onto, or both or neither. T: $R^2 \to R^2$, where T (x, y) = (x + y, x-y)
11.	Determine whether the linear transformation is one-to-one, onto, or both or neither. T: $R^2 \to R^3$, where T (x, y) = (x - y, y - x, 2x-2y)
12.	If $T:V \to W$ is a linear transformation then prove that $T(-v)=-T(v), v \in V$
13.	If $T:V \to W$ is a linear transformation then prove that $T(0)=0, \forall v \in V$
14.	If $T:V \to W$ is a linear transformation then prove that $T(v-w)=T(v)-T(w), \forall v,w \in V$
15.	Prove that if $T:V \to W$ then the range of T is subspace of W
16.	Prove that if $T:V \to W$ then the kernel of T is subspace of V
17.	Prove that identity transformation is linear.
18.	Prove that zero transformation is linear.

19.	Prove that a linear transformation $T:V\to W$ is one-one if nullity(T) = 0
20.	Prove that a linear transformation $T:V\to W$ is one-one if Ker(T) = {0}.
21.	Prove that a linear transformation $T:V\to W$ is one-one if rank(T) = dimV
22.	State and prove the dimension theorem for linear transformation
23.	Prove that if $T_1:U\to V, T_2:V\to W$ are one-one transformation then $T_1\circ T_2$ is also
24.	Prove that if $T_1:U\to V, T_2:V\to W$ are linear transformation then $T_1\circ T_2$ is also linear
[C]	**5 – Marks Questions**
1.	Consider the basis $S=\{v_1,v_2,v_3\}$ for R^3, where $v_1=(1,1,1)$, $v_2=(1,1,0)$ and $v_3=(1,0,0)$ and let $T:R^3\to R^3$ be the linear operator such that $T(v_1)=(2,-1,4)$, $T(v_2)=(3,0,1)$ and $T(v_3)=(-1,5,1)$. Find a formula for $T(x_1,x_2,x_3)$ and use that formula to find $T(2,4,-1)$.
2.	Consider the basis $S=\{v_1,v_2\}$ for R^2, where $v_1=(-2,1)$, $v_2=(1,3)$ and let $T:R^2\to R^3$ be the linear transformation such that $T(v_1)=(-1,2,0)$, $T(v_2)=(0,-3,5)$. Find a formula for $T(x_1,x_2)$ and use that formula to find T(2,-3).
3.	Let $T:P_2\to P_3$ be the linear transformation defined by $T(p(x))=x\,p(x)$. Then Verify dimension theorem
4.	Let $T:R^2\to R^2$ be the linear transformation given by the formula $T(x,y)=(2x-y,-8x+8y)$. (i) Find a basis for $\ker(T)$.(ii) Find a basis for $R(T)$.
5.	Let $T:M_{22}\to M_{22}$ be the linear transformation given by the formula $T\left(\begin{bmatrix} a & b \\ c & d\end{bmatrix}\right)=\left(\begin{bmatrix} a+b & b+c \\ a+d & b+d\end{bmatrix}\right)$. (i) Find a basis for $\ker(T)$.(ii) Find a basis for $R(T)$.
6.	Let $T:R^3\to R^3$ be the linear transformation given by the formula $T(x_1,x_2,x_3,x_4)=(4x_1+x_2-2x_3-3x_4, 2x_1+x_2+x_3-4x_4, 6x_1-9x_3+9x_4)$ Find a basis for $R(T)$
7.	Let $T:R^3\to R^3$ be the linear transformation given by the formula T(x, y, z)=(x-y+z, 2x-z, 2x+3y) then find $T^{-1}(x, y, z)$
8.	Let $T:P_2\to P_2$ be the linear transformation defined by $T(p(x))=p(2x+1)$. Find $[T]_s$ w.r.t the basis $S=\{1,x,x^2\}$

9.	Let $T_1 : M_{22} \to M_{22}$ be the linear transformation defined by $T(A) = A^T$ Let $S_1 = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$ $S_2 = \left\{ \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\}$ be bases for M_{22} ,Then find matrix T w.r.t S_1 and S_2
Unit-3	**Eigen Values and Eigen Vectors**
[A]	**1 – Mark Questions**
1.	Define Eigen values
2.	Define Eigen vectors
3.	Define geometric multiplicity of matrix.
4.	Define Algebric multiplicity of matrix.
5.	State Caylay Hamiton theorem
6.	If the eigen values of matrix A= $\begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix}$ are 1,3 then find the eigen values of matrices[a] 3A [b] A^{-1}
7.	If the eigen values of matrix A= $\begin{bmatrix} 5 & 0 \\ 2 & 1 \end{bmatrix}$ are 1,5 then find the eigen values of following matrices [a] 3A [b] A^T
8.	Write the eigen values of $A = \begin{bmatrix} 3 & 5 & 10 \\ 0 & 14 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
9.	Write the eigen values of $A = \begin{bmatrix} 3 & 5 & 10 \\ 0 & 14 & 0 \\ 0 & 0 & 10 \end{bmatrix}$
10.	Write the eigen values of $A = \begin{bmatrix} 3 & 0 & 0 \\ 11 & 14 & 0 \\ 20 & 10 & 2 \end{bmatrix}$
[B]	**5 – Marks Questions**
1.	Determine algebraic and geometric multiplicity of the matrix $\begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 1 \end{bmatrix}$
2.	Prove that zero is eigen value of matrix iff it is singular.
3.	Prove that the eigen values of a diagonal matrix are the entries on its main diagonal.

4.	Prove that matrix and its transpose have same eigen values.
5.	Prove that λ is an eigen value of matrix A then $k\lambda$ is an eigen value of matrix kA.
6.	If A and P are $n \times n$ matrices such that P is nonsingular matrix and $D = P^{-1}AP$ is diagonal then each column of P is an eigen vector for A.
7.	Find the eigenvalues and eigenvectors for matrix $B = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$
8.	Verify Cayley-Hamilton theorem for the following matrix and hence, find A^{-1} $A=\begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$
9.	Check the following matrix is diagonalizable $A=\begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$
10.	Find A^{10} for $A = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$
11.	Describe the conic C whose equation is $9x^2 + 4y^2 - 36x - 24y + 36 = 0$
12.	Determine the nature , index and signature of Q: $x^2 + 5y^2 + z^2 + 2xy + 2yz + 6xz$
13.	Verify Cayley-Hamilton theorem for the following matrix and hence, find A^{-1} $A=\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$
14.	Determine algebraic and geometric multiplicity of the matrix: $\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$
15.	Find the eigenvalues and eigenvectors for matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$
Unit-4	**Inner Product Space**
[A]	**1 – Mark Questions**
1.	Define inner product space
2.	Define orthogonal projection on a vector
3.	Define distance
4.	Define norm

5.	Define angle
6.	State Cauchy schawarz inequality
7.	State the triangular inequality for norm
8.	Find $\|u\|$ for $u=(-1,2)$
9.	If $u=(3,-2),v=(4,3),k=3$, then find $\langle kv,u\rangle$
10.	If $u=(3,-2),v=(4,3)$ then find $\|u\|$, $\langle v,u\rangle$
11.	If $u=(1,-2),v=(1,3),k=5$ then find $\langle v,ku\rangle$
12.	Find $u=(1,-2),v=(1,3),w=(0,1)$ then find $\langle u,v+w\rangle$
[B]	**3 – Marks Questions**
1.	For inner product space V, prove that $0.u=u.0=0$ for $\forall u\in V$
2.	For inner product space V, prove that $\|u+v\|\le\|u\|+\|v\|$ for $\forall u,v\in V$
3.	For inner product space V, prove that $\langle ku,v\rangle=k\langle u,v\rangle$ for $\forall u,v\in V$
4.	For inner product space V, prove that $\langle u,v\rangle=\langle v,u\rangle$ for $\forall u,v\in V$
5.	State and prove Pythagorean theorem in inner product space.
6.	State and prove triangular inequality for norm in inner product space
7.	Determine the vectors $u=(-4,2,-10,1),v=(0,1,2,9)$ are orthogonal with Euclidean inner product space
8.	Show that $\langle u,v\rangle=9u_1v_1+4u_2v_2$ is and inner product on R^2 generated by matrix A= $\begin{bmatrix}3 & 0\\ 0 & 2\end{bmatrix}$.
9.	Find $d(u,v)$, $\|u\|$ if $u=(1,4),v=(2,-6)$ with stand Euclidean inner product space V.
10.	Verify Cauchy Schwarz inequality for $u=(0,-2,2,1),v=(-1,-1,1,1)$ with Euclidean inner product space V.
11.	Verify Cauchy Schwarz inequality for $u=(-4,2,1),v=(8,-4,-2)$ with Euclidean inner product space V.

12.	Find cosine angle between $u=(-1,5,2), v=(2,4,-9)$ with Euclidean inner product space V.
13.	Let R^3 have the Euclidean inner product. For which values of k, u and v are orthogonal,. where u=(1, 2, 3) , v= (2, k, 6)
14.	Let R^3 have the Euclidean inner product. For which values of k, u and v are orthogonal, where u=(1, 2, 0) , v= (2, k, 6)
15.	Find the orthogonal projection of u along v in R^3 w.r.t Euclidean inner product , where u = (4,0,-1) , v = (3,1,-5)
16.	Find the orthogonal projection of u along v in R^3 w.r.t Euclidean inner product u = (1.-2,3) , v = (1,2,1)
17.	Find the orthogonal projection of u along v in R^3 w.r.t Euclidean inner product u = (4,0,-1) , v = (0,1,-5)
18.	Find the orthogonal projection of u along v in R^3 w.r.t Euclidean inner product u = (1.2,3,4) , v = (1,-3,4,-2)
[C]	**5 – Marks Questions**
1.	Which of the following set of vectors is orthonormal with respect to the Euclidean inner product on R^3? $\left(\frac{2}{3},-\frac{2}{3},\frac{1}{3}\right), \left(\frac{2}{3},\frac{1}{3},-\frac{2}{3}\right), \left(\frac{1}{3},\frac{2}{3},\frac{2}{3}\right)$
2.	Which of the following set of vectors is orthonormal with respect to the Euclidean inner product on R^3? $\left(\frac{1}{5},\frac{1}{5},\frac{1}{5}\right), \left(-\frac{1}{2},\frac{1}{2},0\right), \left(\frac{1}{3},\frac{1}{3},-\frac{2}{3}\right)$
3.	Which of the following set of vectors is orthogonal with respect to the Euclidean inner product on R^3? $\left(\frac{1}{\sqrt{2}},0,\frac{1}{\sqrt{2}}\right), \left(\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},-\frac{1}{\sqrt{3}}\right), \left(-\frac{1}{\sqrt{2}},0,\frac{1}{\sqrt{2}}\right)$
4.	Which of the following set of vectors is orthogonal with respect to the Euclidean inner product on R^3? $(1,0,0), \left(0,\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right), (0,0,1)$
5.	Which of the following set of vectors is orthogonal with respect to the Euclidean inner product on R^3? $(0,1,0,-1), (1,0,1,0), (-1,1,1,1)$
6.	Let R^3 have the Euclidean inner product. Use the Gram-Schmidt process to transform the basis $\{u_1, u_2, u_3\}$ into an orthonormal basis.

	$u_1=(1,0,0),\ u_2=(3,7,-2),\ u_3=(0,4,1)$
7.	Let R^3 have the Euclidean inner product. Use the Gram-Schmidt process to transform the basis $\{u_1, u_2, u_3\}$ into an orthonormal basis. $u_1=(1,2,1),\ u_2=(1,0,1),\ u_3=(3,1,0)$
8.	Let R^2 have the Euclidean inner product. Use the Gram-Schmidt process to transform the basis $\{u_1, u_2\}$ into an orthonormal basis. $u_1=(1,-3),\ u_2=(2,2)$
9.	Let R^3 have the Euclidean inner product. Use the Gram-Schmidt process to transform the basis $\{u_1, u_2, u_3\}$ into an orthonormal basis. $u_1=(1,1,1),\ u_2=(1,-2,1),\ u_3=(1,2,3)$
10.	Find the least squares solution of the linear system Ax=b and find the orthogonal projection of 'b' onto the column space of A. $A=\begin{bmatrix} 1 & 1 \\ -1 & 1 \\ -1 & 2 \end{bmatrix}, b=\begin{bmatrix} 7 \\ 0 \\ -7 \end{bmatrix}$
11.	Find the least squares solution of the linear system Ax=b and find the orthogonal projection of 'b' onto the column space of A. $A=\begin{bmatrix} 1 & -1 \\ 3 & 2 \\ -2 & 4 \end{bmatrix}, b=\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}$