Semester – II : 60090205 - CC3 Advanced Calculus

Question Bank

Unit-1	Partial differentiation
[A]	**5 – Marks Questions**
1.	Explain the following terms: a) Limit for functions of two variables. b) Continuous function of two variables.
2.	Explain the following terms: a) Partial derivative. b) Homogenous function of partial differentiation
3.	Find the limits of the following: a) $\lim_{(x,y)\to(2,1)} \frac{xy}{x^2+y^2}$ b) $\lim_{(x,y)\to(3,4)} \sqrt{x^2+y^2-1}$
4.	Check whether the given limit is exists or not $\lim_{(x,y,z)\to(0,0,0)} \frac{xy+yz^2+xz^2}{x^2+y^2+z^2}$
5.	Check whether the given functions are homogenous or not? a) $u = f(x,y) = x^3 + y^3 + 2x^2y$ b) $u = f(x,y) = (y^2+4xy)^{\frac{3}{2}}$
6.	Write the statement of Modified Euler's theorem. Write the statement of Euler's theorem.
7.	If $f(x,y) = e^{x+y+z}$ find $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$
8.	If $f(p,q) = \tan^{-1}\frac{q}{p}$ find $\frac{\partial f}{\partial p}, \frac{\partial f}{\partial q}$
9.	Find F_x, F_y for $F(x,y,z) = 1 + xy^2 - 2z$
10.	Find the value of $\frac{\partial f}{\partial x}$ at the point $(4,-5)$ if $f(x,y) = x^2 + 3xy + y - 1$.
11.	If resistor of R_1, R_2 and R_3 ohms are connected in parallel to make on R ohm resistor, the value of R can be found the equation $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$ find the value of $\frac{\partial R}{\partial R_2}$ when $R_1 = 30, R_2 = 45, R_3 = 90$ ohms.
12.	Express $\frac{dz}{du}, \frac{dz}{dv}$ as function of u and v both by using chain rule also evaluate $\frac{dz}{du}$ and $\frac{dz}{dv}$ at the given point (u,v). $z = 4e^x \log y$, where $x = \log(u\cos v)$ and $y = u\sin v$
13.	If $u = \log(x^2+y^2)$ prove that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$.
14.	If $u = x^3 + y^3 + z^3 + 3xyz$, find (i) f_x , (ii)f_{xx} , (iii)f_{xyz}
15.	Prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$ for $u = \tan^{-1}\left(\frac{x}{y}\right)$

16.	If $u = x^3 + y^3 - 3axy$, prove that $\frac{\partial^2 u}{\partial x\,\partial y} = \frac{\partial^2 u}{\partial y\,\partial x}$
17.	Find f_{yxyz} if $f(x,y,z) = 1 - 2xy^2z + x^2y$.
18.	If $u = \frac{x^2+y^2}{x+y}$ then prove that $\left(\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right)^2 = 4\left(1 - \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right)$
19.	If $u = (x^2 + y^2 + z^2)^{-\frac{1}{2}}$ prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = -u$
20.	If $u = f(r)$ where $r^2 = x^2 + y^2$, prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f^{II}(r) + \frac{1}{r}f^{I}(r)$
21.	If $u = \log(x^2 + y^2 + z^2)$, show that $x\frac{\partial^2 u}{\partial y\,\partial z} = y\frac{\partial^2 u}{\partial z\,\partial x}$
22.	If $u = x^2y + y^2z + z^2x$, prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 2(x + y + z)$
23.	Find the value of $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at the point (1,1,1) if $z^3 - xy + yz + y^3 - 2 = 0$
24.	If $w = \log(x^2 + y^2 + z^2)$ where $x = \cos t$, $y = \sin t$, $z = 4\sqrt{t}$ .Find $\frac{dw}{dt}$ at the point t = 3.
25.	Verify Euler's theorem for a function $u = \sin^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x}$.
26.	Verify Euler's theorem for a function $u = (\sqrt{x} + \sqrt{y})(x^n + y^n)$
27.	If $u = \frac{x^2+y^2}{\sqrt{x+y}}$ then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial x} = \frac{3}{2}u$
28.	If $u = \frac{1}{x^2} + \frac{1}{xy} + \frac{\log x - \log y}{x^2+y^2}$ then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = -2u$
29.	If $u = \cos\left(\frac{xy+yz+zx}{x^2+y^2+z^2}\right)$ then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$
30.	If $u = \left(\frac{x}{y} + \frac{y}{x} + \frac{z}{x}\right)^n$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$
31.	If $u = x^2y\sin^{-1}\left(\frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}}\right)$then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3u$
32.	If $u = x^4y\sin^{-1}\frac{x}{y}$, find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.
33.	If $u = x^2\tan^{-1}\left(\frac{y}{x}\right) + y^2\sin^{-1}\left(\frac{x}{y}\right)$ then prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 2u$
34.	If $u = x + y + (x + y)f\left(\frac{y}{x}\right)$ then prove that $x\left(\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial x\,\partial y}\right) = y\left(\frac{\partial^2 u}{\partial y^2} - \frac{\partial^2 u}{\partial x\,\partial y}\right)$
35.	If $u = x^n f\left(\frac{y}{x}\right) + y^{-n}\emptyset\left(\frac{x}{y}\right)$ then $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} + x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = n^2u.$
36.	If $x = e^u\tan v$ and $y = e^u\sec v$ then find the value of $\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right)\cdot\left(x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y}\right)$
37.	$u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ show that $\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right) = \sin 2u$

38.	If $u = \sin^{-1}\left(\frac{x^{\frac{1}{4}}+y^{\frac{1}{4}}}{x^{\frac{1}{5}}+y^{\frac{1}{5}}}\right)$ then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{20}\tan u$
39.	If $u = \mathrm{cosec}^{-1}\left(\frac{x+y}{x^2+y^2}\right)$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$
40.	If $u = \sin^{-1}\left(\frac{x^{\frac{1}{4}}+y^{\frac{1}{4}}}{x^{\frac{1}{6}}+y^{\frac{1}{6}}}\right)$ then prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{1}{144}\tan(\tan^2 u - 1)$
41.	$u = \tan^{-1}(x^2 + 2y^2)$ show that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 2\cos 3u \sin u$
42.	State and prove the Euler's theorem
43.	State and prove the Modified Euler's theorem
44.	State and prove the Taylor's theorem.
45.	Expand e^{ax} sin by in power of x and y.
46.	Expand $e^x\cos y$ in power of $(x-1)$and $\left(y-\frac{\pi}{4}\right)$.
47.	Expand $x^2y + 3y - 2$ in the neighborhood of the point $(1,-2)$.
48.	Expand $\sin(x+h)(y+k)$ by Taylor's series.
Unit-2	**Applications of Partial Differentiation**
[A]	**5 – Marks Questions**
1.	Explain the following terms: Tangent plane Normal line
2.	Explain the following terms: Jacobian Absolute maximum
3.	Explain the following terms: Absolute minimum Strict maximum
4.	Write the formula of tangent plane Write the formula of normal line.
5.	Find the tangent plane and normal line to the surface $f(x,y,z) = x^2 + y^2 + z - 9 = 0$ a circular parabolied at the point $p_0(1,2,4)$.
6.	Find the tangent plane and normal line at P_0 on the given surface (1) $x^2 - 4y^2 + 3z^2 + 4 = 0$ at the point $(3,2,1)$ (2)$\cos\pi x - x^2y + e^{xz} + yz = 4$ at the point $(1,1,-1)$
7.	If $f(x,y,z) = x^2y + 2xz^2 = 8$ at point $(3,2,1)$
8.	If measurement of radius of base and height of a right circular cone are incorrect by -1% and 2% , prove that there is no error in the volume.
9.	The period of a simple pendulum is given by $T = 2\pi\sqrt{\frac{l}{g}}$. If Tis found using $l = 8$ ft., $g = 32$ ft/sec^2 , find an approximate error in T if the correct values are

	$l = 8.05\text{ ft.}, g = 32.01\text{ ft/sec}^2$
10.	A balloon is in the shape of a right circular cylinder of radius 3m and length 6m, and is surmounted by hemispherical ends. If the radius is increased by 1% m and the length by 5% m, find the percentage increase in the volume of a balloon.
11.	Obtain the percentage error in the area of an ellipse when an error of 1 percentage is made in measuring the semi major and semi minor axes.
12.	The H.P. required to propel a steamer varies as the cube of the velocity and the square of length. If there is 3% increase in velocity and 4% increase in length, find the corresponding percentage increase in H.P.
13.	If $x = r\cos\theta$ and $y = r\sin\theta$, find $\frac{\partial(x,y)}{\partial(r,\theta)}$ and $\frac{\partial(r,\theta)}{\partial(x,y)}$
14.	If $u = \frac{x+y}{1-xy}$ and $v = \tan^{-1}x + \tan^{-1}y$, find $\frac{\partial(u,v)}{\partial(x,y)}$.
15.	If $x = r\sin\theta\cos\emptyset$, $y = r\sin\theta\sin\emptyset$, $z = r\cos\theta$, find $\frac{\partial(x,y,z)}{\partial(r,\theta,\emptyset)}$.
16.	If $u = 2xy$, $v = x^2 - y^2$, $x = r\cos\theta$, $y = r\sin\theta$ find $\frac{\partial(u,v)}{\partial(x,y)} \cdot \frac{\partial(x,y)}{\partial(r,\theta)}$
17.	If $x = e^u \sec v$, $y = e^u \tan v$ then , prove that $J \cdot J^I = 1$.
18.	If $x = v^2 + w^2, y = w^2 + u^2, z = u^2 + v^2$, prove that $J \cdot J^I = 1$.
19.	If $u = xyz$, $v = x^2 + y^2 + z^2$, $w = x + y + z$, find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.
20.	If $x = \sqrt{vw}, y = \sqrt{wu}, z = \sqrt{uv}$ and $u = r\sin\theta\cos\emptyset$, $v = r\sin\theta\sin\emptyset$, $w = r\cos\theta$, find $\frac{\partial(x,y,z)}{\partial(r,\theta,\emptyset)}$
21.	If $x = e^u \sec v, y = e^y \tan v$ then prove that $J \cdot J^I = 1$
22.	If $u = 2xy, v = x^2 - y^2$ and $x = r\cos\theta, y = r\sin\theta$ find $\frac{\partial(u,v)}{\partial(r,\theta)}$
23.	Find the stationary value of $x^3 + y^3 - 3axy$, $a > 0$. Also find the extremum values.
24.	Find the extreme value of $x^2 + y^2 + 6x + 12$.
25.	Find the extreme value of $x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$.
26.	For $f(x,y) = x^3 + y^3 - 3xy$, examine the extremum values.
27.	Find the maximum and minimum values of $x^3 + y^3 - 63(x + y) + 12xy$. $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$. $x^2y^2 - 5x^2 - 8xy - 5y^2$.
Unit-3	**Vectors Functions**
[A]	**1 – Mark Questions**
1.	Define tangent vector
2.	Define normal plane
3.	Define unit tangent vector.
4.	Define normal vector.
5.	Define binormal vector.
6.	Define field

7.	Define Scalar point function
8.	Define vector point function.
9.	Define gradient.
10.	Define curvature
11.	Define osculating plane
12.	Define Torsion
[B]	**3 – Marks Questions**
1.	If $\bar{r} = a(\sin\omega t)\,\hat{\imath} + b(\sin\omega t)\,\hat{\jmath} + \frac{ct}{\omega^2}(\sin\omega t)\,\hat{k}$, prove that $\frac{d^2\bar{r}}{dt^2} + \omega^2\bar{r} = \frac{2c}{\omega}(\cos\omega t)\hat{k}$.
2.	Verify the $\frac{d}{dt}(\bar{A}\times\bar{B}) = \frac{d\bar{A}}{dt}\times\bar{B} + \bar{A}\times\frac{d\bar{B}}{dt}$ for $\bar{A} = 5t^2\hat{\imath} + t\hat{\jmath} - t^3\hat{k}$ *and* $\bar{B} = \sin t\,\hat{\imath} - cost\hat{\jmath}$
3.	Find unit tangent, unit normal and unit binormal vectors for the curve $x = t, y = 3\sin t\,, z = 3\cos t$.
4.	Find the length of the curve $\bar{r}(t) = 2t\hat{\imath} + 3\sin 2t\hat{\jmath} + 3\cos 2t\hat{k}$ on the interval $0 \le t \le 2\pi$
5.	A particle moves on the curve $x = 2t^2, y = t^2 - 4t, z = 3t - 5$. Find the velocity at t=1 in the direction of $\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$.
6.	Find unit tangent, unit normal and unit binormal vectors for the curve $x = a\cos\theta\,, y = a\sin\theta\,, z = b\theta$.
7.	If $\bar{r} = t^3\hat{\imath} + \left(2t^3 - \frac{1}{5t^2}\right)\hat{\jmath}$, then show that $\bar{r}\times\frac{d\bar{r}}{dt} = \hat{k}$
8.	Find unit tangent, unit normal and unit binormal vectors for the curve $\bar{r} = 2cost\hat{\imath} + 3\sin t\hat{\jmath} + 4t\hat{k}$ at $t = \pi$.
9.	If $\bar{r} = \bar{a}\sin ht + \bar{b}\cos ht$, where $\bar{a}$ *and* $\bar{b}$ are constant vectors, then show that (1) $\frac{d^2\bar{r}}{dt^2} = \bar{r}$, (2)$\frac{d\bar{r}}{dt}\times\frac{d^2\bar{r}}{dt^2} = constant$.
10.	Find the magnitude of the velocity and acceleration of a particle which moves along the curve $x = 2\sin 3t\,, y = 2\cos 3t\,, z = 8t$ at any time $t > 0$. Find unit tangent vector to the curve.
11.	If $\bar{a}$ *and* $\bar{b}$ are constant vectors and ω is constant and $\bar{r} = \bar{a}\sin\omega t + \bar{b}\cos\omega t$, prove that $\bar{r}\times\frac{d\bar{r}}{dt} + \omega(\bar{a}\times\bar{b}) = 0$.
12.	Find the length of the curve $\bar{r}(t) = 5t\hat{\imath} + 2\sin 2t\hat{\jmath} + 2\cos 2t\hat{k}$ on the interval $0 \le t \le 2\pi$
13.	If $\bar{r} = \bar{a}\sin ht + \bar{b}\cos ht$, where $\bar{a}$ *and* $\bar{b}$ are constant vectors, then show that (1) $\frac{d^2\bar{r}}{dt^2} = \bar{r}$, (2)$\frac{d\bar{r}}{dt}\times\frac{d^2\bar{r}}{dt^2} = constant$.
[C]	**5 – Marks Questions**
1.	If $\bar{r} = (a\cos t)\,\hat{\imath} + (a\sin t)\,\hat{\jmath} + (at\tan\alpha)\,\hat{k}$, prove that (1) $\left\lvert\frac{d\bar{r}}{dt}\times\frac{d^2\bar{r}}{dt^2}\right\rvert = a^2 sec\alpha$ (2) $\left[\frac{d\bar{r}}{dt}\cdot\frac{d^2\bar{r}}{dt^2}\cdot\frac{d^3\bar{r}}{dt^3}\right] = a^3\tan\alpha$
2.	For the curve $\bar{r} = acos\theta\hat{\imath} + a\sin\theta\hat{\jmath} + b\theta\hat{k}$, find the radius of curvature and torsion.
3.	Find the radius of curvature and torsion for the curve $x = t^2 - 1, y = t^3 - 1, z = t^4 - 1$ at $t = 1$.

4.	Find the curvature and torsion for the curve $\bar{r} = \cos t\hat{\imath} + \sin t\hat{\jmath} + t\hat{k}$. Also prove that $2(k^2 + \tau^2) = 1$.
5.	Find the curvature and torsion for the $x = tcost\ , y = tsint, z = \lambda t\ at\ t = 0$
6.	Find $\nabla\emptyset\ at\ (1,-2,1), if\ \emptyset = 3x^2y - y^3z^2$
7.	Evaluate ∇e^{r^2}, where $r^2 = x^2 + y^2 + z^2$.
8.	Find the unit vector normal to the surface $x^2 + y^2 + z^2 = a^2$ at $\left(\frac{a}{\sqrt{3}}, \frac{a}{\sqrt{3}}, \frac{a}{\sqrt{3}}\right)$
9.	Find the angle between the normal to the surface $xy = z^2$ at $P(1,1,1) and\ Q(4,1,2)$.
10.	The temperature of the points in space is given by $\emptyset = x^2 + y^2 - z$. A mosquito located at point (1,1,2) desires to fly in such a direction that it will get warm as soon as possible. In what direction should it move?
11.	If $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, prove that $div(grad\ r^n) = n(n+1)r^{n-2}$
12.	Prove that for vector function $\bar{A}$, $\nabla \times (\nabla \times \bar{A}) = \nabla(\nabla \cdot \bar{A}) - \nabla^2\bar{A}$.
13.	If $\bar{A} = \nabla(xy + yz + zx)$, find $\nabla \cdot \bar{A}$ and $\nabla \times \bar{A}$.
14.	Verify, $\nabla \times (\nabla \times \bar{A}) = \nabla(\nabla \cdot \bar{A}) - \nabla^2\bar{A}$ for $\bar{A} = x^2y\hat{\imath} + x^3y^2\hat{\jmath} - 3x^2z^2\hat{k}$.
15.	Find the curl of $\bar{A} = e^{xyz}(\hat{\imath} + \hat{\jmath} + \hat{k})$ at the point (1,2,3).
16.	Find curl $\bar{A} = x^2y\hat{\imath} - 2xz\hat{\jmath} + 2yz\hat{k}$ at the point (1,0,2).
17.	Determine the constants a and b such that curl of $(2xy + 3yz)\hat{\imath} + (x^2 + axz - 4z^2)\hat{\jmath} + (3xy + 2byz)\hat{k}$ is zero.
18.	Verify, If f, g are scalars and $\bar{A}\ and\ \bar{B}$ are vectors, then (1) $\nabla(fg) = f\nabla g + g\nabla f$ (2) $\nabla \times (\bar{A} \times \bar{B}) = (\bar{B} \cdot \nabla)\bar{A} - \bar{B}(\nabla \cdot \bar{A}) - (\bar{A} \cdot \nabla)\bar{B} + \bar{A}(\overline{\nabla \cdot B})$
19.	Prove that $\nabla^2\left[\nabla\left(\frac{\bar{r}}{r^2}\right)\right] = 2r^{-4}$, where $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$.
20.	Prove that $\nabla\left(\nabla \cdot \frac{\bar{r}}{r}\right) = -\frac{2}{r^3}\bar{r}$.
21.	Show that $\bar{E} = \frac{\bar{r}}{r^2}$ is irrotational.
22.	Calculate $\nabla^2 f$ when $f = 3x^2z - y^2z^3 + 4x^3y + 2x - 3y - 5$ at the point (1,1,0).
23.	Prove that $\nabla\left(r\nabla\frac{1}{r^n}\right) = \frac{n(n-2)}{r^{n+1}}$.
24.	Prove that $\nabla^2 f(r) = \frac{d^2f}{dr^2} + \frac{2}{r}\frac{df}{dr}$, $where\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$.
Unit-4	**Vector calculus and its Applications**
[A]	**5 – Marks Questions**
1.	State stokes' theorem.
2.	State Green's theorem.
3.	State Gauss' theorem.
4.	Evaluate $\int_c \bar{F}d\bar{r}$ where $\bar{F} = (3x^2 + 6y)\hat{\imath} - 14yz\hat{\jmath} + 20xz^2\hat{k}$ and C is the straight line joining the points $(0,0,0)\ to\ (1,1,1)$.
5.	Find the work done in moving a particle from $A(1,0,1)\ to\ B(2,1,2)$ along the straight line AB in the force field $\bar{F} = x^2\hat{\imath} + (x-y)\hat{\jmath} + (y+z)\hat{k}$.
6.	Evaluate $\iint_s \bar{F}\hat{n}ds$, where $\bar{F} = 18z\hat{\imath} - 12\hat{\jmath} + 3y\hat{k}$ and S is the part of the plane

	$2x + 3y + 6z = 12$ in the first octant.
7.	Evaluate $\iint_s (yzdydz + xzdzdx + xydxdy)$ over the surface of the sphere $x^2 + y^2 + z^2 = 1$ in the positive octant.
8.	Evaluate $\iiint_V \bar{F} dV$ where $\bar{F} = x\hat{\imath} + y\hat{\jmath} + 2z\hat{k}$ and V is the volume enclosed by the planes $x = 0, y = 0, x = a, y = a, z = b^2 and\ the\ surface\ z = x^2$.
9.	Evaluate $\iiint_V (\nabla \times \bar{F}) dV$, where $\bar{F} = (2x^2 - 3z)\hat{\imath} - 2xy\hat{\jmath} - 4x\hat{k}$ and V is the closed region bounded by the plane $x = 0, y = 0, z = 0\ and\ 2x + 2y + z = 4$.
10.	Evaluate $\int_c \bar{F} d\bar{r}$ along the parabola $y^2 = x$ between the points (0,0) and (1,1) where $\bar{F} = x^2\hat{\imath} + xy\hat{\jmath}$.
11.	Verify Green's theorem for $\oint[(x^2 - 2xy)dx + (x^2y + 3)dy]$ where C is the boundary of the region bounded by the parabola $y = x^2$ and the line $y = x$.
12.	Verify Green's theorem for $\oint[(y - sinx)dx + cosxdy]$ where C is the plane triangle enclosed by the lines $y = 0, x = \frac{\pi}{2}, y = \frac{2x}{\pi}$.
13.	Verify stokes' theorem for the vector field $\bar{F} = (x^2 - y^2)\hat{\imath} + 2xy\hat{\jmath}$ in the rectangular region in the xy-plane bounded by the lines $x = -a, x = a, y = 0, y = b$.
14.	Verify stokes' theorem for $\bar{F} = (x + y)\hat{\imath} + (y + z)\hat{\jmath} - x\hat{k}$ and S is the surface of the plane $2x + y + z = 2$ which is in the first octant.
15.	Verify Gauss' divergence theorem for $\bar{F} = 4xz\hat{\imath} - y^2\hat{\jmath} + yz\hat{k}$ over the cube $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$
16.	Verify Gauss' divergence theorem for $\bar{F} = 2x^2y\hat{\imath} - y^2\hat{\jmath} + 4xz^2\hat{k}$ over the region bounded by the cylinder $y^2 + z^2 = 9$ and the plane $x = 2$ in the first octant.